Determination of the laminar burning velocity and the Markstein length of powder–air flames

A.E. Dahoe a,b,*, K. Hanjalic c, B. Scarlett a

*Department of Chemical Engineering, Delft University of Technology, Julianalaan 136, 2628 BL Delft, The Netherlands
bDepartment of Engineering, University of Cambridge, Cambridge, United Kingdom
cDepartment of Applied Physics, Delft University of Technology, Julianalaan 136, 2628 BL Delft, The Netherlands

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Abstract

This work deals with the determination of the laminar burning velocity and introduces the Markstein length of powder–air mixtures. A powder burner was used to stabilize laminar cornstarch–air dust flames and the laminar burning velocity was determined by means of laser Doppler anemometry. The dust concentration was varied from 0.26 to 0.38 kg m⁻³. The measured laminar burning velocities were found to be sensitive to the shape of the flame. With the same dust concentration, parabolic flames were found to have a laminar burning velocity, which was almost twice that of a planar flame ca. 30 cm s⁻¹ for the latter as compared with ca. 54 cm s⁻¹ for the former. From this discrepancy and the flame curvature, the Markstein length could be determined. It was found to have a value of 11.0 mm. This Markstein length was subsequently used to correct the measured laminar burning velocities at various dust concentrations in order to obtain the unstretched laminar burning velocity. The unstretched laminar burning velocity lies between 15 and 30 cm s⁻¹ and is thought to be a property of the dust and of the concentration. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The severity of a dust explosion is a function of a wide variety of parameters. Some of these parameters represent properties that are specific to the chemical properties of the exploding dust–air mixture, while others reflect the sensitivity of the explosion to the flow properties of the dust cloud. Due to the absence of a comprehensive description of the transient combustion behavior of any particular dust–air mixture under arbitrary conditions (pressure, temperature, flow properties), it is common practice to measure the explosion severity in laboratory test vessels (1-m³ vessel, 20-l sphere) and to predict what would happen if the same mixture exploded in an industrial plant unit. This is currently done by means of the well-known cube-root-law [1–3]. The maximum rate of pressure rise measured in the test vessel is multiplied by the cube root of the test volume to yield a $K_{St}$ value, which is assumed to be a volume invariant dust explosion severity index. The dust explosion severity of the same mixture in a plant unit is predicted by dividing this $K_{St}$ value by the cube root of the volume of the plant unit. The resulting dust explosion severity dictates the design basis for safety protection (e.g. explosion relief venting, explosion suppression). In this approach, known as the VDI methodology, it is assumed that laboratory test data can be considered to be applicable to accidental explosions in plant units and to represent a conservative case even when the actual industrial circumstances are not reproduced in the laboratory experiments.

Already from the beginning, the VDI methodology was questioned by a number of researchers. Eckhoff [4] pointed out that the cube-root-law was no more than an approximation of a single realization of the explosion pressure curve. Based on earlier work by Nagy et al. [5] on the pressure development during the course of explosions in spherical vessels, he discussed the conditions under which the cube-root-law is a valid approximation of the dust explosion severity, and the conditions under which the cube-root-law may be used to scale dust explosion severities, measured in laboratory test vessels, into dust explosion severities that one might expect in larger industrial vessels.

First, the mass burning rate (i.e. the product of the burning velocity, the flame area, and the density of the unburnt mixture which is to be consumed by the flame)
has to be the same in both the test vessel and the industrial vessel at the moment when the rate of pressure rise reaches its maximum value. This condition is only fulfilled when both vessels are spherical, ignition occurs at the center of both vessels, changes in the pressure and temperature of the unburnt mixture ahead of the flame have the same effect on the burning velocity, and the turbulent flow fields, as well as changes in the flow fields during the course of an explosion, are identical in both vessels. In practice none of these requirements are fulfilled because the changes in the pressure and temperature of the unburnt mixture are not the same in differently sized vessels containing identical mixtures, and dust explosion severity testing is done in laboratory vessels under conditions of significant but unknown turbulence. Moreover, test results from laboratory vessels are applied to industrial situations with unknown turbulence. Based upon the fact that dust explosion severity increases with increasing turbulence intensity, it is presently assumed that if the turbulence intensity in the laboratory test vessels is made high enough, laboratory test results yield conservative estimates of what may happen during an explosion in a plant unit. This, however, may lead to unacceptable over-estimations in situations where the turbulence levels in industrial practice are much lower than those created in laboratory test vessels, but also to under estimations of the explosion severity under circumstances where additional turbulence is generated by the explosion itself. Tamanini et al. [6] demonstrated that worst case predictions by means of the VDI methodology may underestimate the dust explosion severity when the turbulence varies at the time of the explosion.

Secondly, the thickness of the flame must be negligible with respect to the radius of the vessel. An inherent flaw of the cube-root-law is that it does not take the phenomenon of flame thickness into account. As the flame thickness becomes appreciable with respect to the apparatus radius (i.e. > 1%), the cube-root-law becomes increasingly inaccurate in predicting the maximum rate of pressure rise [7]. If for example chemically identical mixtures, under physically identical conditions, are ignited to deflagration in two differently sized vessels, application of the cube-root-law to the maximum rate of pressure rise, measured in the two vessels yields different $K_{so}$ values. Since many powders have a flame thickness that is appreciable with respect to the radius of the 20-l sphere, and even with respect to the radius of the 1-m$^3$ vessel, the cube-root-law may not be considered as generally valid for the prediction of dust explosion severity.

In order to overcome the limitations associated with the cube-root-law, several models have been proposed by other researchers. Unlike the cube-root-law, which takes the single instant of the rate of pressure rise measured in a test vessel to predict a single instant of the rate of pressure rise during an industrial explosion, these so-called integral balance models are capable of predicting the entire pressure evolution during an explosion. More importantly, since their derivation is based on fundamental relationships between the pressure development and the mass burning rate at any instant, the effect of mixture composition, pressure, temperature, and turbulence on the transient combustion process can be taken into account explicitly. Existing models of this kind are those of Bradley and Mitchell [8], Nagy and Verakis [9], Perlee et al. [10], and Chirila et al. [11], Bradley et al. [12], Tamanini [13] and Dahoe et al. [7]. Ideally, these models enable engineers to predict the behavior of explosions, and hence the explosion severity, under industrial circumstances when the behavior of the mass burning rate is known from laboratory experiments and the (varying) turbulent aerodynamic parameters are known for the industrial circumstances. In the case of premixed gases, the burning velocity and the flame thickness are recognized as fundamental measures of the driving force behind the combustion process and these quantities have been used with success to model the mass burning rate.

When burning velocities and flame thickness are used as key parameters in integral balance models, a distinction is made between a laminar burning velocity and a laminar flame thickness, on the one hand, and a turbulent burning velocity and a turbulent flame thickness on the other. The reason for making this distinction is that, when a flame is stabilized in a laminar flow of combustibles, it establishes itself at a fixed position in the flow field and its surface remains smooth. In other words, it inherits the laminar behavior of the flow field. Moreover, the velocity at which the cold reactants enter the flame zone in the normal direction, the laminar burning velocity, appears to be a mixture specific property. It reflects the sensitivity of the combustion process to changes in the chemical composition, fuel concentration, oxygen content, particle size, pressure and temperature of the approaching flow of reactants. By contrast, when a flame is trapped within a turbulent flow of combustibles, it inherits the turbulent nature of the flow field: the turbulence of the approaching flow continuously distorts the flame and ceaselessly shifts its position in space between certain geometrical boundaries. As a result, the surface area of the instantaneous laminar flame changes in a chaotic manner, which is determined by the turbulence of the flow field. Owing to the fact that the relevant time scales of the fluid structures that compose the turbulent flow field are much larger than the chemical time scale of the instantaneous combustion zone, the geometrical boundaries between which the instantaneous flame front shifts its position are identified as a turbulent flame thickness. Due to the enhancement of heat and mass transfer by turbulence, the turbulent flame zone propagates with a turbulent burning velocity which is greater than the laminar burning velocity. The local consumption of reactants at any particular portion of the flame surface, however, occurs within a zone whose width is equal to the local laminar flame thickness and at a rate which is determined by the local laminar burning velocity. It is for
this reason that the turbulent burning velocity and the turbulent flame thickness are always expressed in terms of a combination of the laminar flame propagation parameters and the turbulence features of the flow field (i.e. root-mean-square velocity and length scale).

While the laminar burning velocity and the laminar flame thickness of a premixed gas are considered as fundamental mixture properties, their application to dust clouds appears to be controversial. This controversy stems from the fact that, while on a macroscopic scale, a dust cloud may seem like a homogeneous mixture of fuel and oxidizer, on a microscopic scale it consists of a number of discrete particles immersed in a continuum of the oxidizer. With solid fuel particles of a density of 1000 kg m\(^{-3}\), and a particle size of 15 \(\mu\)m, one finds an interparticle spacing of about 150 \(\mu\)m when the solid fuel concentration is equal to 500 g m\(^{-3}\). With the interparticle spacing being 5–10 times the particle size (dust explosions typically involve dust concentrations between 125 and 1000 g m\(^{-3}\)), a combustible dust cloud consists of a number of discrete particles that are separated by a distance, which are orders of magnitude larger than the molecular mean-free path. As a result, physical and chemical properties that determine the magnitude of the laminar burning velocity may not be considered as volume average mixture properties.

Bradley and Lee [14] pointed out that dust clouds do possess a laminar burning velocity and a laminar flame thickness similar to the ones used in the modeling of gas flames if the fuel particles emit appreciable amounts of volatile components. As the particle temperature increases in the preheat zone of the flame, volatile components begin to be emitted and mix with the surrounding oxidizer. This leads to the formation of a primary reaction zone that is largely sustained by gaseous combustion. There are the heat release in the primary reaction zone and the subsequent conduction of heat into the unburnt mixture that determine the magnitude of the laminar flame propagation parameters. In comparison with premixed gases, the laminar burning velocity and the laminar flame thickness of a dust cloud additionally depend on the rate of evolution of volatile components, the mixing of these volatile components with the oxidizer that surrounds the particles, the coupling of the particle and gas phase oxidation and the radiative energy exchange between the flame and the unburnt mixture.

The emission of volatile components and the mixing process continues into the reaction zone and is largely completed before the end of the reaction zone. With lignites, bituminous coals, vegetable grains and powdered foodstuffs, the remaining char burns in the hot oxygen beyond the preheat zone. Particles of less than 1 \(\mu\)m can be oxidized entirely in the primary reaction zone and the effect of the chemical heat release of the oxidation of these particles on the laminar burning velocity is additive to that of the volatile species. For particle sizes greater than 10 \(\mu\)m, the oxidation of char in the primary reaction zone is slow in comparison with the gaseous combustion process. In this case, the oxidation of char within the primary reaction zone hardly affects the laminar burning velocity. However, when the oxygen concentration beyond the primary reaction zone is high enough, the subsequent oxidation of the char particles may result in the formation of a secondary reaction zone. The presence of a secondary reaction zone where the combustion of char occurs may increase the laminar burning velocity because of the additional heat release. When gas phase reactions have such a predominant influence on the laminar flame propagation of dust clouds, then it is furthermore anticipated that the enhancement of this burning velocity due to the effects of turbulence is similar to what occurs with entirely gaseous premixed flames [15]. In spite of the similarity with the combustion of premixed gases and the importance of laminar flame propagation parameters in the modeling of dust explosions, our knowledge of these flame characteristics lags behind in what is already known for gas flames.

It is the purpose of this paper to demonstrate that the same approach which has been successful in the study of gas flames can also be applied to gas–solid mixtures. Hence, the present study deals with the determination of the laminar burning velocity of dust–air mixtures. Cornstarch was chosen as the model material because of its consistent composition and particle size, its high volatile content, and its use by previous researchers [15,16]. In this experimental work, laminar cornstarch flames were stabilized by means of a powder burner and the laminar burning velocity at the center of the flame surfaces was determined by performing velocity measurements in the flow field along the symmetry axis. The velocity measurements were achieved by means of laser Doppler anemometry. The laminar burning velocity of cornstarch–air mixtures was determined with dust concentrations from 0.26 to 0.38 kg m\(^{-3}\). The determination of the unstretched laminar burning velocities from the measured laminar burning velocities requires the use of an additional parameter, namely, the Markstein length. The role and significance of this quantity will be clarified in the next section.

The laminar burning velocity of cornstarch–air mixtures was measured by previous researchers by means of other methods. The results obtained in this work are compared with the results obtained by two previous investigations namely, those of Proust and Veyssiere [16] and Bradley et al. [15].

2. The laminar burning velocity and the Markstein length

In order to clarify the aim and structure of the present study of cornstarch–air mixtures, it is helpful to consider the determination of the laminar burning velocity of premixed gases.

A common way to measure the laminar burning velocity of premixed gases is by means of a Bunsen burner (see
The device essentially consists of a tube that serves as a mixing chamber for the fuel and the oxidizer. When the combustible mixture is ignited, a stationary flame surface establishes itself at a small distance above the tube. The detachment of the flame is caused by the fact that the heat losses to the rim sustain a narrow region where the temperature is so low that combustion cannot occur. The flame anchors itself in that position by adapting its shape to changes in the velocity of the oncoming mixture. Depending on the magnitude and the spatial distribution of the exit velocity of the unburnt mixture, the flame may assume a variety of shapes (see Fig. 1). If the exit velocity, $v$, is less than twice the laminar burning velocity, the flame will have a parabolic shape. At greater exit velocities, the flame assumes a conical shape and at still higher velocities (e.g., five times the laminar burning velocity) the shape becomes hyperbolical and ‘tip-blowthrough’ may occur.

Burning velocity measurements with the Bunsen burner are often conducted with conical flames. The method relies on a relationship between the laminar burning velocity of a conical flame and the flow speed at the burner exit. Strictly speaking, each point of the flame surface has a radius of curvature in the direction normal to the unburnt mixture. However, if the conical flame is sufficiently large, a surface element of the cone mantle may be regarded locally as an oblique planar combustion wave as depicted by Fig. 2. The dashed line represents a surface element of the boundary where the temperature rises just above the initial temperature of the unburnt mixture, $T_u$, due to heat conduction from the combustion wave. The associated density change beyond the dashed line is indicated by the deflected streamlines. In the case of a planar flame the streamlines intersect the flame perpendicularly and the separation between any two adjacent streamlines remains constant along the flame surface provided that the increment of the corresponding stream function is the same. In other words, the tangential velocity to the surface element, $v_{t,i}$, is zero and does not change along the flame surface, and the unburnt mixture enters the flame perpendicularly with a uniform velocity $v_{n,i}$. The laminar burning velocity of the surface element may therefore be related to the velocity at the tube exit by measuring the cone angle $\alpha$ and by decomposing the exit velocity $v$ along the dashed line into a normal and a tangential component ($v_n$ and $v_t$). Then,

$$\rho_0 S_{ul} = \rho_0 v_n = \rho_0 v \sin \alpha = \rho_0 v_{t,i},$$

and hence

$$S_{ul} = v \sin \alpha.$$  (2)

However, when this methodology is applied to the center of the flame tip, and there is nothing wrong in doing so, it is found that $S_{ul} = v$ because $\alpha = 90^\circ$. Thus, the laminar burning velocity at the center of the flame tip is found to be a factor $1/\sin \alpha$ greater than that of the cone mantle and it is evident that different laminar burning velocities exist within the same combustible mixture. This implies that the laminar burning velocity may not be regarded as a fundamental mixture property unless it is ‘normalized with respect to flame shape’ in some way. This issue has been addressed by a number of researchers [17–31].

The increased laminar burning velocity at the center of the flame tip is associated with two phenomena namely, flame stretch and flame curvature. The flame tip is a narrow region where the cone mantle develops curvature. As the oncoming unburnt mixture reaches the flame tip, it is heated by the lateral parts of the flame, and the uniform velocity profile is therefore distorted. The unburnt mixture no longer enters the flame perpendicularly and has a tangential velocity component. The tangential velocity also
has a gradient along the flame surface. A laminar flame within a nonuniform flow field with a sufficiently strong velocity gradient along its surface is subjected to tangential strain and therefore develops curvature. If a curved flame is concave with respect to the unburnt mixture (upper part of Fig. 3), adjacent points traveling along the flame surface move closer together (negative flame stretch) in the tangential direction. Conversely, if the flame is convex towards the unburnt mixture (lower part of Fig. 3), adjacent points on the flame surface move further apart (flame stretch) in the tangential direction. The convergence and the divergence of the streamlines along the flame surface in Fig. 3 illustrate how the local flame structure is being modified by the strain. It is evident that the mass flow per unit area of a concave flame exceeds that of a planar flame and for a convex flame it becomes less.

Although flame curvature is invariably coupled to nonuniformities of the velocity field, it must be realized that even in the absence of strain, it may significantly alter the local laminar burning velocity. A flame surface serves as a local sink for reactants and a local source for heat. Increasing the diffusion rate of the reactants to the flame increases the rate of heat release and hence, tends to increase the flame temperature. An increase of the conductive heat flux from the flame sheet into the oncoming unburnt mixture tends to reduce the flame temperature. However, changes in the conductive heat flux are coupled to changes in the diffusive flux of the reactants towards the flame surface. When a planar flame develops curvature, the heat conducted from one location is convected to another location at the flame surface. A convex bulge conducts heat into the oncoming unburnt mixture, as shown in the lower part of Fig. 4, and this heat is convected towards parts of the bulge away from the center. This leads to an increased laminar burning velocity at the lateral parts of the flame and to a lower laminar burning velocity at the center. In the case of concavity, shown in the upper part of Fig. 4, the heat conducted by the lateral parts of the bulge is convected towards the center and the effect on the laminar burning velocity is reversed.

In the case of weakly strained flames, the influence of flame stretch on the laminar burning velocity of a curved flame is taken into account by expressing the latter as follows [17,p.22,32,p.357],

$$S_{\text{al}} = \left[ 1 + \frac{\mathcal{L}}{\mathcal{R}} \right] S_{\text{al}}^0.$$  \hspace{1cm} (3)

Here $S_{\text{al}}^0$ denotes the laminar burning velocity of an unstretched flame and the quantity $\mathcal{L}$ is known as the Markstein length. $\mathcal{R}$ is the radius of curvature of the flame sheet, defined as the reciprocal of the mean curvature [33,p.136]. It is taken to be positive for convexity towards the burnt mixture and negative for convexity towards the unburnt mixture.

The Markstein length, $\mathcal{L}$, introduced by Clavin [21] for gaseous fuels, is a mixture specific constant with a magnitude of the order of the flame thickness and serves as a measure of the sensitivity of the laminar burning velocity to the influence of flame shape modifications. Neither its theoretical nor its experimental evaluation is easy, and much remains to be learned about its precise functional dependence on the chemical and transport properties of a specific mixture. It is for this reason that the Williams–Clavin formula is currently accepted as an adequate relationship to be used in conjunction with Eq. (3) in order to describe the response of the laminar burning velocity to changes of the flame shape. The Williams–Clavin formula [20,23,34] is given by,

$$MK \equiv \frac{\mathcal{L}}{\delta_{\text{L}}} = \frac{1}{\gamma} \ln \left( \frac{1}{1-\gamma} \right) + \frac{Ze(Le - 1)(1-\gamma)}{2\gamma} \times \int_0^{1/(1-\gamma)} \ln \left( 1 + \frac{1}{x} \right) \frac{dx}{x}.$$  \hspace{1cm} (4)

This formula was derived for a two reactant mixture with a single-step overall reaction rate, a large activation energy, a constant thermal conductivity, a constant kinematic viscosity, and a constant specific heat. In this equation $MK \equiv \mathcal{L}/\delta_{\text{L}}$ denotes the Markstein number, $Ze \equiv (E_a / RT_f^2)(T_i - T_u)$ the Zeldovich number, $Le \equiv \lambda / C_p ID$ the Lewis number, $\lambda$ the thermal conductivity, $C_p$ the specific heat at constant pressure, $\delta_{\text{L}}$ the laminar flame thickness, $T_i$ the adiabatic flame temperature, $T_u$ the unburnt mixture temperature, $E_a$ the activation energy, and $RT_f^2$ the reaction rate. The flame stretch, $\mathcal{L}$, is defined as the reciprocal of the mean curvature of the flame sheet. The Zeldovich number, $Ze$, and the Lewis number, $Le$, are related to the Lewis number, $Le$, and the Zeldovich number, $Ze$, by

$$Ze = \frac{E_a}{RT_f^2} \frac{\lambda}{C_p I D}$$

and

$$Le = \frac{\lambda}{C_p ID}$$

respectively. The Zeldovich number, $Ze$, and the Lewis number, $Le$, are related to the Lewis number, $Le$, and the Zeldovich number, $Ze$, by

$$Ze = \frac{E_a}{RT_f^2} \frac{\lambda}{C_p I D}$$

and

$$Le = \frac{\lambda}{C_p ID}$$

respectively.
number and \( \gamma = (T_i - T_e)/T_e \). With \( \gamma \) typically lying between 0.8 and 0.9, this formula predicts that \( L \) is 2.0–2.6 times the thickness of an unstretched laminar flame, \( \delta_0 \), if \( Le = 1 \). More sophisticated expressions have also been developed. Clavin and Garcia [28,35] for example, extended Eq. (4) to include the temperature dependence of the thermal and molecular diffusivity. Rogg and Peters [31] derived an analytical expression similar to Eq. (4) by performing a theoretical analysis on a weakly strained stoichiometric methane–air flame using a reduced three-step mechanism with six reactants. They rejected the simplification of a global single step reaction involving only two reactants, but retained the assumption that the diffusivities are independent of the temperature.

When laminar flames are subjected to severe stretch, Eq. (3) is no longer valid and one must resort to an equation that invokes two Markstein lengths: one for strain, \( L_s \) and one for curvature effects, \( L_c \). The stretch rate, \( \dot{s} \), of a surface element, \( A \), in a strained fluid is defined as,

\[
\dot{s} = \frac{1}{A} \frac{dA}{dt}.
\]

(5)

It consists of two additive contributions, \( \dot{s}_s \) and \( \dot{s}_c \). The first includes the influence of strain and the second involves the effect of curvature. At any point of a flame surface, these contributions are related to the velocity of the flow field, \( \nu \), and the normal unit vector of the surface in the direction of the unburnt mixture by the universal expression,

\[
\dot{s} = -nn \cdot \nabla \nu + \nabla \cdot \nu + \frac{S_{ul} \nabla \cdot n}{\dot{s}_c}.
\]

(6)

Here \( S_{ul} \) is the magnitude of the unstretched laminar burning velocity vector. Clavin [21] has shown that when a planar laminar flame with a thickness \( \delta_0 \) is distorted into a bulge of a size \( A \gg \delta_0 \), then the local laminar burning velocity at each point can be related to the local stretch rate as,

\[
\frac{S_{ul} - S_{ul}}{S_{ul}} = \frac{L_s}{S_{ul}} \left( \frac{1}{A} \frac{dA}{dt} \right) + \sigma(\epsilon^2).
\]

(7)

Here \( \epsilon = \delta_0/A \) is a small number. This equation can be rewritten into,

\[
S_{ul}^0 - S_{ul} = \frac{L_s}{S_{ul}} \left( \frac{1}{A} \frac{dA}{dt} \right) + \sigma(\epsilon^2).
\]

(8)

\[
S_{ul}^0 - S_{ul} = L_s + \sigma(\epsilon^2).
\]

(9)

It is then obvious that the Markstein length is intended to serve as a proportionality ratio between the change of the laminar burning velocity and the stretch rate. In order to reconcile this result with the general expression for the stretch rate Eq. (6), researchers have decided to express \( L_s \) as a linear combination of quantities that account for
the separate effect of variables such as strain rate, flame curvature, and pressure, each having its own Markstein length. Thus, \( L_s \) was replaced by \( L_s + L_c \) and hence,

\[
S_{nl} - S_{al} = (L_s + L_c) + \sigma(e^2). \tag{10}
\]

Combination of Eqs. (6), (8) and (10) leads to the equation which is to be used in case of severe flame stretching,

\[
S_{nl} = L_c[nn\nabla v - \nabla \cdot v] + \left[1 - L_c \nabla \cdot n\right]S_{nl} \tag{11}
\]

\[
S_{al} = L_c[nn\nabla v - \nabla \cdot v] + \left[1 + (\kappa_1 + \kappa_2) L_c\right]S_{al}. \tag{12}
\]

\[
S_{nl} = L_c[nn\nabla v - \nabla \cdot v] + \left[1 + \frac{L_c}{A}\right]S_{al}. \tag{13}
\]

It is seen that Eq. (13) may be simplified to Eq. (3) when the influence of strain is negligible. In the step from Eq. (12) to Eq. (13), use has been made of the relationship (see Equations (9.38.7) and (9.41.8) of Ref. [36]),

\[
\nabla \cdot n = -(\kappa_1 + \kappa_2). \tag{14}
\]

Here \( \kappa_1 \) and \( \kappa_2 \) denote the minimum and maximum curvature of an arbitrary surface. The mean curvature is defined as the sum of these quantities and its reciprocal value is considered to be the radius of curvature, \( A \), of a flame surface.

The Markstein length of several gaseous fuels was investigated by a number of researchers and the most illustrative of these investigations will be mentioned here. Quinard [21,28] used a wrinkled flame burner to experimentally determine the Markstein length. The study involved hydrogen–, methane–, ethylene–, and propane–air mixtures as seen in Fig. 5. The measurement technique will be described here to illustrate how the Markstein length was determined. The wrinkled flame burner, shown in Fig. 6, consists of three sections: a settling chamber, a convergent section and a rectangular burner head. The combustible mixture is made laminar in the settling chamber, and an initial uniform velocity profile at the entrance of the convergent section is ensured by grids. At the exit of the convergent section there is an array of water cooled tubes. These tubes cause laminar perturbations in the velocity profile and as a result, the flame takes the form of a two-dimensional sinusoidal sheet. At the exit of the burner, a water-cooled grid decouples the hot combustion products from the cold ambient air. The curvature of the flame front was obtained by means of a photo-diode array and by fitting a portion of the digitized picture to a sinusoid of an appropriate wavelength. The local burning velocity was measured by means of laser doppler anemometry. The curvature and velocity measurements were performed at

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**Fig. 8.** Markstein numbers of methane–air mixtures from numerical experiments from Bradley et al. [24].

**Fig. 9.** The powder burner set-up used to measure the laminar burning velocity of cornstarch–air mixtures by means of laser Doppler anemometry.
the minima and maxima of the flame surface where the
strain and curvature systematically have opposite signs and
equal magnitudes. Here, also the tangential component of
the flow velocity is zero and practically no horizontal
velocity gradient exists.

Bradley et al. [24] solved the conservation equations for
mass, momentum, species and energy for inwardly ori-
ented spherical laminar methane–air flames, initiated by an
instantaneous ignition around an outer spherical boundary.
The reduced kinetic mechanism adopted was that of Mauss
and Peters (C_1) with 18 species and 40 reactions, in which
O\cdot, HO\cdot, HO_2\cdot, H_2O_2, \cdotCH, \cdotCH_2, \cdotCHO, \cdotCH_2O,
and \cdotCH_3 are steady-state species. As these ‘numerical
implosion experiments’ resolved the details of the flow
field and the curvature at any particular location of the
flame front, the separate Markstein lengths, \mathcal{L}_e and \mathcal{L}_c,
could be determined over a wide range of equivalence
ratios. These quantities and the associated Markstein num-
bers are shown in Figs. 7 and 8.

3. Experimental set-up

The experimental set-up is shown in Fig. 9 and consists
of two main parts: a powder burner to create stabilized
dust flames and a laser Doppler anemometer for measuring
the flow velocity at various locations inside the flame
zone.

The powder burner consists of a glass tube in which the
combustible particles are fluidized together with a certain
amount of glass beads. Before each experiment the tube
was filled with a mixture of approximately 100 g of 44–88
\mu m glass beads and about 300 g of cornstarch. The tube
has a length of 51.5 cm, an internal diameter of 4.8 cm and
a burner head, with a 28.5-mm internal diameter burner
rim, is mounted onto the tube exit. The distributor is a
porous plate and the flow rate is regulated by means of a
flow meter. Since the pressure drop across the fluidized
bed is impossible to predict exactly, and since the flow
meter is calibrated at atmospheric pressure, it cannot be
used to control the flow rate. Hence, the exact flow rate
had to be calculated for each flame from the velocities at
the tube exit as measured with the laser Doppler anemome-
ter (see Fig. 10).

Figs. 11 and 12 show the radial profile of the measured
vertical velocity component at two different heights above
the burner head, namely, at a distance of 2.5 and 6.3 mm.
Unlike the parabolic profiles commonly observed at the
exit of the Bunsen burner, the velocity profiles at the
powder burner exit were observed to be flat. The reason
for this is that the dust cloud enters the burner head as a
laminar, plug flow due to the uniform velocity distribution
created by the fluid bed and the length to diameter ratio of
the burner head is too small (diameter 28.5 mm, length 55
mm) for a parabolic profile to be formed. In the case of the
Bunsen burner, the length to diameter ratio is sufficiently
large to allow the development of Poiseuille flow.

At greater heights, there is a zone where the initially
uniform velocity distribution shows a change that makes it
necessary to consider two possible causes. The first possible cause is the behavior of a pressure gradient. Inside the burner head, the pressure gradient is oriented along the symmetry axis. As soon as the fluid exits the burner head, it expands and the pressure gradient points away from center line. As a result, the flow changes its direction in the outer parts of the jet, and the vertical velocity component becomes smaller.

The second possible cause may be sought in the momentum diffusion, away from the laminar jet and into the ambient air due to the shear forces. The distance across which this alters the velocity profile can be estimated by means of,

$$
\delta = \sqrt{\pi \nu t},
$$

where $\delta$ denotes the penetration depth, $\nu$ the kinematic viscosity and $t$ the time during which the momentum transfer occurs. At an exit velocity of 30 cm s$^{-1}$, a stationary fluid element needs about 0.02 s to cross a distance of 6.3 mm. With these values and a kinematic viscosity of $10^{-5}$ m$^2$ s$^{-1}$, one finds a penetration depth of 0.8 mm, which does not account for the observed distortion of the velocity profile at a height of 6.3 mm. At larger heights, however, this effect would become an important factor and, together with the effect of the pressure gradient, the laminar jet may entrain ambient air into the flame.

The laser Doppler anemometer used in this work measures only one velocity component and consists of the following components: a laser, a beam splitter, a photomultiplier tube, a high voltage power supply and a correlator. The laser, the beam splitter and the photomultiplier tube are mounted on a type RF 340 optical bench, supplied by Malvern Instruments. The laser is a Melles Griot type 05-LHP-927 He–Ne laser with an output power of 50 mW. It emits a light beam with a wavelength of 632.8 nm and obtains its energy from a Melles Griot 05-LPL-944-080 power supply.
Table 1

Optimal values of the parameters of Eq. (16)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value ± S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$(11.5 ± 2.0) \times 10^{-1}$</td>
</tr>
<tr>
<td>$v$</td>
<td>$(970.1 ± 3.8) \times 10^{-3}$</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>$(20.6 ± 7.5) \times 10^{-3}$</td>
</tr>
<tr>
<td>$m$</td>
<td>$(23.8 ± 2.5) \times 10^{-2}$</td>
</tr>
<tr>
<td>$r$</td>
<td>$(92.9 ± 9.8) \times 10^{-6}$</td>
</tr>
<tr>
<td>$s$</td>
<td>$7.919 \times 10^{-6}$ (fixed)</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$(-1.8 ± 2.0) \times 10^{-1}$</td>
</tr>
</tbody>
</table>

For a 95% confidence interval, multiply S.E. by 1.96.

The primary laser beam is divided into two equal intensity beams by means of the beam splitter, which can be rotated around an axis that coincides with the center line of the primary beam. The separation between the two new beams can be varied by adjusting the position and the orientation of the optical crystals inside the beam splitter. In this work, the separation between the two beams was set to 20 mm and a converging lens with a focal length of 250 mm was used to cross the beams in order to form the probe volume. Hence, the fringe spacing was equal to $7.919 \times 10^{-6}$ m. When a particle crosses the probe volume, it scatters the light of both incident beams, and the scattered light is collected in the backscatter mode by an EMI 9863 KB/100 photomultiplier. Before reaching the photomultiplier, the scattered light passes through a zoom-lens, a 200-μm pinhole and an optical filter that admits light of narrow range around 632.8 nm. The high voltage (optimal signals were obtained in the voltage range between 1.2 and 2.5 kV) was supplied by an EMI PM 28B power supply. The photomultiplier tube converts the optical signal into a time series of TTL signals with a repetitive pattern of which the cycle time corresponds to the modulation frequency of the optical signal. The electrical signals given off by the photomultiplier are subsequently processed by a 64-channel Malvern K7025 correlator, which computes the autocorrelation function of the received electrical signals.

It can be derived [37] that the autocorrelation function, $G(\tau)$, has the following form,

$$G(\tau) = a_1 \exp \left( -\frac{v^2 \tau^2}{r^2} \right) \left[ 1 + \frac{m^2}{2} \right]$$

$$\times \exp \left( -\frac{2(\pi \sigma_r \tau)^2}{s^2} \right) \cos \left( \frac{2\pi v \tau}{s} \right) + b_1. \quad (16)$$

In this equation: $a_1$ denotes a scaling factor; $v$ denotes the velocity component of the scattering particle, perpendicular to the bisection of the angle formed by the crossing laser beams; $\sigma_r$ denotes the standard deviation of $v$; $\tau$ denotes the separation time; $m$ denotes the fringe visibility; $r$ denotes the effective radius of the measuring volume; $s$ denotes the fringe spacing; and $b_1$ denotes the contribution of the background signal.

The correlator was continuously operated in a specific program, named. This program collects the autocorrelation functions and performs a rudimentary form of data validation before storage in a digital computer.

- The program normalizes the autocorrelation function on the basis of the highest recorded value and checks whether the highest normalized value occurs in the first few channels of the correlator.

- The program verifies that the normalized autocorrelation function has a decreasing trend.

At each measuring location in the flow field, at least six valid autocorrelation functions were gathered, and the velocity was determined by fitting Eq. (16) to these data sets. Fig. 13 shows an example of a fitted autocorrelation function, and the optimal values of the parameters of Eq. (16) are shown in Table 1.

4. Determination of the laminar burning velocity and the Markstein length

Deshaies and Cambray [38] performed an experimental study on the laminar burning velocity of propane–oxygen–nitrogen mixtures with a nonuniform flow field. The experimental set-up used by these authors is shown in Fig. 14. The reactive mixtures flowed from a cylindrical tube, where different kinds of honeycomb structures and damping grids are positioned and impinges on a flat stagnation surface. After ignition, axi-symmetric flames with a positive, negative and zero curvature were seen to stabilize below the stagnation plate. Laser Doppler velocimetry was used to measure the local velocity of the fluid and laser tomography to visualize the shape of a meridian section of the flame front. The visualization of the flame front was
accomplished by seeding the reactive mixture with small oil droplets that evaporate upon entering the hot zones of the flame. Prior to entering the flame zone, these droplets serve as scatterers of the laser sheet light, and after evaporation there are no scatterers present. The meridian section of the flame, which appears as a transition region between the bright unburnt mixture and the dark flame zone, was recorded by means of a video camera and the pictures were digitized and stored as grayscale images.

Deshais and Cambray distinguished four zones in the flow field of the flames (Fig. 15): the flow field of the unburnt mixture which is not influenced by the heat of the flame, the preheat zone of the flame, the reaction zone, and a final zone with the combustion products. In the first zone, the vertical velocity component was found to decrease owing to the presence of the stagnation plate at a downstream position in the flow field. The decrease of the vertical velocity component continues until the heat of the
flame accelerates the fluid. This minimum velocity marks the upstream boundary of the preheat zone. Deshaies and Cambray fitted a straight line to the velocity profile in the first zone and extrapolated this information to what the velocity would be, in the absence of heat effects, at the position where the flame temperature attains its highest value. This velocity was taken as the laminar burning velocity by these authors.

In the present work, eight cornstarch dust flames were investigated and these are specified here as flames A–H. At low flow rates (flames A–C), the dust flames were observed to stabilize as a flat flame, very close to the burner rim. At higher flow rates (flames E–H) the flame was found to stabilize further away from the burner rim and its shape changed into a parabola. Between these extremes there seems to be more than one stable situation (flame D): the flame showed a tendency to change its shape continually between the parabolic shape and the planar shape, with a preference for the former (moderately curved). The associated mass throughput and particle velocities measured with the laser Doppler anemometer are shown in Fig. 16, and it is seen that this transitional behavior of the flame occurs when the exit velocity at the burner head is equal to approximately 0.5–0.55 m s⁻¹. Fig. 17 shows the height of the flame tip above the burner head of the flames investigated.

Figs. 18 and 19 show the profiles of the vertical velocity component measured in flames (C) and (D). With the dust flames it is also possible to discern a flow field structure as depicted by Fig. 17, and it is interesting to see that the flow velocity increases further in the after-burning zone. This indicates that the laminar combustion process of cornstarch–air mixtures may involve multiple flame zones. Only the first combustion zone, the actual flame, is the truly premixed flame. The after-burning zone is considered to be a diffusion flame where the burning of char takes place, as mentioned in the introduction of this paper and pointed out by Bradley and Lee [14].

The laminar burning velocity of the dust flames was taken to be equal to the minimum velocity that marks the upstream boundary of the preheat zone for two reasons. The first reason is that an extrapolation like the one applied by Deshaies and Cambray would lead to unacceptably low and even negative values. The second reason is
that it is desirable to relate the propagation velocity of the flame to known properties of the combustible mixture involved. Since the mixture properties undergo changes in the preheat zone that cannot be described by means of a simple extrapolation from the mixture properties prior to the preheat zone, such an extrapolation leads to an undefined, and perhaps nonexistent, frame of reference. In order to find the minimum value of the vertical velocity component and to overcome the errors introduced by the scatter in the measured data, a third order polynomial was fitted to the initial part of the velocity profiles [39] and the resulting laminar burning velocities are shown in Fig. 20.

The laminar burning velocities shown in Fig. 20 appear to be clustered according to the shape of the flame front. The more the shape deviates from a planar flame towards a parabolic flame, the higher the experimental burning velocity becomes. At a dust concentration of 0.33 kg m$^{-3}$ the laminar burning velocity is observed to increase by a factor of about two when the flame shape changes from planar to parabolic. The reasons for this behavior are the flame curvature and strain due to nonuniformities in the velocity field which were discussed in Section 2. Eq. (13) was used to find the value of the unstretched laminar burning velocity.

Since the pressure gradient at the burner exit is small and there is no stagnation point in the flow field, the contribution of the first term on the right hand side of the above equation is considered to make a negligible contribution to the experimental value of the laminar burning velocity. Hence, the equation may be simplified to Eq. (3), taking only the effect of the flame curvature into account.

The first step in using Eq. (17) to find the unstretched laminar burning velocity is to find the Markstein length, $L_M$, from the experimental data at a dust concentration of 0.33 kg m$^{-3}$ in Fig. 20. The idea behind this correction is that, since the dust concentration is the same in flames B, C and G, there is a Markstein length such that Eq. (17) must give the same unstretched laminar burning velocity.
Table 2
The laminar burning velocity $S_{ul}$, the radius of curvature $\rho$ and the unstretched laminar burning velocity $S_{ul}'$ of flames A–H

<table>
<thead>
<tr>
<th>Flame</th>
<th>Dust conc. (kg m$^{-3}$)</th>
<th>$S_{ul}$ (cm s$^{-1}$)</th>
<th>$\rho$ (m)</th>
<th>$S_{ul}'$ (cm s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.26</td>
<td>22.8</td>
<td>0.3125</td>
<td>22.0</td>
</tr>
<tr>
<td>B</td>
<td>0.33</td>
<td>29.4</td>
<td>0.5</td>
<td>28.8</td>
</tr>
<tr>
<td>C</td>
<td>0.33</td>
<td>30.0</td>
<td>2.5</td>
<td>29.0</td>
</tr>
<tr>
<td>D</td>
<td>0.29</td>
<td>38.0</td>
<td>3.11 × 10$^{-2}$</td>
<td>28.1</td>
</tr>
<tr>
<td>E</td>
<td>0.21</td>
<td>33.1</td>
<td>1.12 × 10$^{-2}$</td>
<td>16.7</td>
</tr>
<tr>
<td>F</td>
<td>0.27</td>
<td>41.4</td>
<td>8.7 × 10$^{-1}$</td>
<td>18.3</td>
</tr>
<tr>
<td>G</td>
<td>0.33</td>
<td>54.5</td>
<td>1.35 × 10$^{-2}$</td>
<td>30.0</td>
</tr>
<tr>
<td>H</td>
<td>0.38</td>
<td>48.8</td>
<td>7.3 × 10$^{-1}$</td>
<td>19.5</td>
</tr>
</tbody>
</table>

The data band reported by these authors was obtained by photographing the flame and by employing the parametric form of the parabola as

$$1 = \frac{y''(s)}{1 + y''^2(s)}^{3/2}.$$

Application of this procedure and Eq. (17) to obtain the same unstretched laminar burning velocity, $S_{ul}'$, for flames (B) and (G) resulted in a Markstein length, $L_M$, of 11.0 mm. Although the Markstein length is known to depend on the composition of the combustible mixture, the latter value was subsequently used to calculate the unstretched laminar burning velocity of all flames, and the results are plotted in Fig. 21. The unstretched laminar burning velocities are between 15 and 30 cm s$^{-1}$.

Fig. 22 shows a comparison between the unstretched laminar burning velocities obtained by means of the powder burner and the results of Proust and Veyssiere [16]. The data band reported by these authors was obtained by observing the motion of upwardly propagating laminar cornstarch–air flames in a tube. The parabolic dust flames were allowed to propagate from the open end of the tube (at the bottom) up to the closed end (at the top) in quiescent cornstarch–air mixtures of various dust concentrations. The laminar burning velocity was taken as the difference between the velocity of the flame contour and the velocity of the unburnt mixture ahead of it, obtained from photographic records of the moving flame. None of the resulting laminar burning velocities was corrected for flame stretch and flame curvature.

Although our results and those of Proust and Veyssiere appear to be of the same magnitude, an interpretation of Fig. 22 in terms of flame stretch and flame curvature puts things in a different perspective. It was mentioned in Section 2 that when a flame front bulges convexly with respect to the unburnt mixture, it propagates with a lower burning velocity than that of a planar flame. The moving flames studied by Proust and Veyssiere were parabolic and convex with respect to the unburnt mixture (see the photographs presented by these authors [16]). A correction of the laminar burning velocity of a parabolic flame by means of the radius of curvature and the Markstein length according to Eq. (3) changes the measured laminar burning velocity by a factor of 1.5 to 2 (see flames (D–H) in Table 2). In their case the associated unstretched laminar burning velocities would be between 40 and 50 cm s$^{-1}$. This is about twice the unstretched laminar burning velocities obtained in our experiments using the powder burner. The reason for this discrepancy must be sought in the fact that buoyancy may have a different effect on moving flames in a tube than in the case of a stationary flame. When a flame propagates in the upward direction from the open end of a tube, the flame enters the unburnt mixture at the downstream side with a velocity that is assisted by the buoyancy force. As a result, the velocity difference between the moving flame and the moving reactants ahead of it, which is the measured burning velocity, may be larger than in the case of the powder burner, where the unburnt mixture enters a stationary flame from the downstream side.

Bradley et al. [15] determined the laminar burning velocity of a 0.260 kg m$^{-3}$ cornstarch–air mixture by igniting turbulent dust clouds to deflagration in a fan-stirred bomb and found a laminar burning velocity of 12 cm s$^{-1}$. The turbulent burning velocity was determined at various, known, turbulence levels. Next, the ratio between the turbulent burning velocity and the laminar burning velocity, $S_{ul}'/S_{ul}$, was plotted against the ratio between the root-mean-square value of the turbulent velocity fluctuations and the laminar burning velocity, $v_{rms}/S_{ul}$. From this plot, the laminar burning velocity could be determined by means of extrapolation to a zero turbulence level (i.e. $v_{rms} \rightarrow 0$). This extrapolation is based on the assumption that the generalized correlation for the turbulent burning velocity of gaseous fuels,

$$\frac{S_{ul}'}{S_{ul}} = 1 + C \left( \frac{v_{rms}}{S_{ul}} \right)^n,$$

is also applicable to the turbulent burning velocity of dust clouds. In this equation, $C$ is a parameter that depends on the length scale of the turbulence and $n$ is known as the bending exponent. For stoichiometric propane–air mix-
Fig. 21. The unstretched laminar burning velocity, $S_{o\text{u}}$, of flames A to H.

Fig. 22. A comparison between the unstretched laminar burning velocity, $S_{o\text{u}}$, of flames A–H and measurements by Proust and Veyssiere [16].

A dust concentration of 0.260 kg m$^{-3}$, Bradley et al. found a laminar burning velocity (12 cm s$^{-1}$) which is about half the unstretched laminar burning velocity obtained by means of the powder burner (22 cm s$^{-1}$; see Table 2). This difference can be attributed to the fact that the results obtained by means of the fan-stirred bomb were not corrected for the effect of flame stretch and flame curvature. The turbulent burning velocities used in the extrapolation were obtained at a flame radii of 20, 25, 30, and 35 mm. As the flames grow in the outward direction after point ignition, they are convex with respect to the unburnt mixture and, as discussed in Section 2, the laminar burning velocity is less than that of a planar flame. When these flame radii are compared with the radii of curvature in Table 2, it is seen that a correction by means of the Markstein length leads to an unstretched laminar burning velocity of about 1.5 times the measured laminar burning velocity. With this correction, one obtains an unstretched laminar burning velocity of 18 cm s$^{-1}$ which is closer to the value of 22 cm s$^{-1}$ found with the powder burner.
5. Conclusions

- Laminar burning velocities of cornstarch–air mixtures of various dust concentrations (0.26–0.38 kg m⁻³) have been measured by stabilizing laminar dust flames on a powder burner. The measured laminar burning velocity was found to be sensitive to the shape of the flame. When the dust concentration was kept the same, parabolic flames were found to have a laminar burning velocity which was almost twice the laminar burning velocity of a planar flame (ca. 30 cm s⁻¹ for the latter as compared with ca. 54 cm s⁻¹ for the former; see Fig. 20 and Table 2).

- From the discrepancy mentioned under the previous item the flame curvature Mark–Stein length, $\mathcal{L}$, of laminar cornstarch–air flames could be determined, and it was found to have a value of 11.0 mm. With this Markstein length, the measured laminar burning velocities at various dust concentrations could be corrected in order to obtain the unstretched laminar burning velocity of cornstarch–air mixtures as a function of the dust concentration (see Fig. 21). The unstretched laminar burning velocities are between 15 and 30 cm s⁻¹.

- The development of damaging pressures due to explosions is predicted by means of integral balance models. With gas explosions in particular, the turbulent burning rate is commonly incorporated by means of Eq. (18) or a similar expression and it is evident that inaccuracies in the laminar burning velocity invariably result in large errors. Since the Markstein length of cornstarch–air mixtures is much larger than the Markstein length of methane–air mixtures (11.0 mm for the former as compared with 0.1, 0.2 mm for the latter; see Fig. 7) errors due to the use of laminar burning velocities which are not corrected by means of the Markstein length are more severe for dust–air mixtures than for purely gaseous premixtures. Hence, knowledge of the Markstein length is of crucial importance in the modeling of dust explosions.

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Flame area (m²)</td>
</tr>
<tr>
<td>$C_P$</td>
<td>Constant pressure specific heat (J mol⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>$E_a$</td>
<td>Activation energy (J mol⁻¹)</td>
</tr>
<tr>
<td>$K_{st}$</td>
<td>Volume normalized maximum rate of pressure rise of a dust explosion (Pa m s⁻¹)</td>
</tr>
<tr>
<td>$n$</td>
<td>Bending exponent in generalized turbulent burning velocity correlation (–)</td>
</tr>
<tr>
<td>$R$</td>
<td>Universal gas constant (J mol⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>$\dot{s}$</td>
<td>Stretch rate (s⁻¹)</td>
</tr>
<tr>
<td>$\dot{s}_t$</td>
<td>Stretch rate due to strain (s⁻¹)</td>
</tr>
<tr>
<td>$\dot{s}_c$</td>
<td>Stretch rate due to curvature (s⁻¹)</td>
</tr>
<tr>
<td>$S_{sl}$</td>
<td>Laminar burning velocity (m s⁻¹)</td>
</tr>
<tr>
<td>$S_{tr}$</td>
<td>Turbulent burning velocity (m s⁻¹)</td>
</tr>
<tr>
<td>$T$</td>
<td>Absolute temperature (K)</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Flame temperature (K)</td>
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<tr>
<td>$T_e$</td>
<td>Temperature of the unburnt mixture (K)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Velocity vector (m s⁻¹)</td>
</tr>
<tr>
<td>$\nu_{rms}$</td>
<td>Root-mean-square value of the velocity fluctuations (m s⁻¹)</td>
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Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$\gamma$</td>
<td>Heat capacity ratio (–)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Penetration depth (m)</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>Laminar flame thickness (m)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity (m² s⁻¹)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Thermal conductivity (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density (kg m⁻³)</td>
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Other symbols

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>ID</td>
<td>Diffusion coefficient (m² s⁻¹)</td>
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<tr>
<td>$\mathcal{L}$</td>
<td>Markstein length (m)</td>
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<td>Strain Markstein length (m)</td>
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<tr>
<td>$\mathcal{L}_r$</td>
<td>Curvature Markstein length (m)</td>
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<td>$\mathcal{R}$</td>
<td>Radius of flame curvature (m)</td>
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Dimensionless groups

<table>
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<td>$Le$</td>
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<tr>
<td>$Mk$</td>
<td>Markstein number (–)</td>
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<td>$Ze$</td>
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References