REACTING FLUID DYNAMICS (2)

PHENOMENA IN THE HYDROGEN-AIR BOUNDARY OR MIXING LAYER

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Topics

- **HEAT AND MASS TRANSFER**

Transfer Based on Molecular Exchange

Heat Transfer; Mass Transfer; Explanation of Transfer Phenomena on the Basis of Gas Dynamics;

Transfer Based on Turbulent Exchange

Characteristics of Turbulent Flow; Mixing Length Theory; $k-\varepsilon$ ***(Two Equations) Model***

Thermophoresis, Soret Effect, and Dufour Effect

Thermophoresis; Soret Effect; Dufour Effect

- **TEMPERATURE AND CONCENTRATION PROFILES**

Variation of Temperature and Species Concentration in a Quiescent Atmosphere

Variation of Temperature Profile; Variation of Species Concentration Profile

Topics (Continued)

Variation of Temperature and Species Concentration in a Laminar Flow

Temperature and Species Concentration Profiles in the Boundary Layer; Temperature and Species Concentration Profiles in a Jet; Temperature and Species Concentration Profiles in Parallel Flows of Different Velocities

Temperature and Species Concentration Profiles in Turbulent Flows

Turbulent Diffusion at the Site with Temperature or Species Concentration Difference; Temperature and Species Concentration Profiles across a Turbulent Jet; Temperature and Species Concentration Profiles across a Shear Flow

- **STRUCTURE OF REACTION SITES**

Surface Reaction

Catalytic Reaction; Combustion Reaction;

Reaction in a Gas Flow

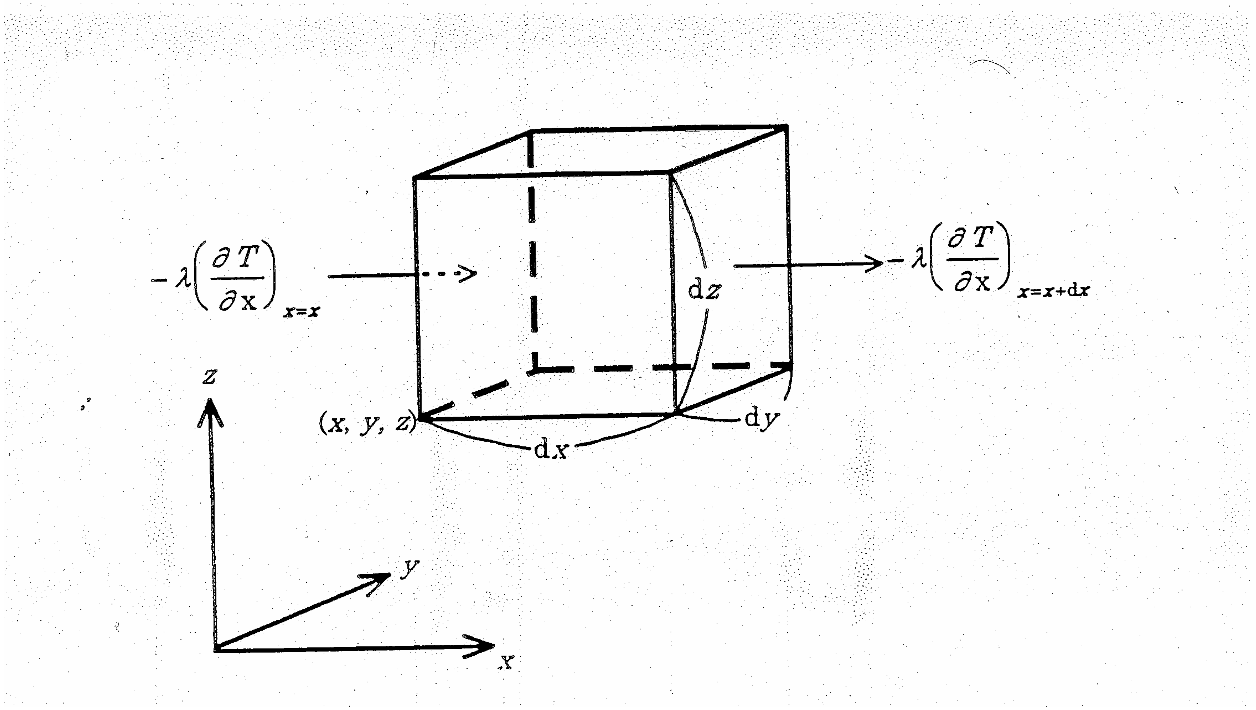
Steady Premixed Flame; Diffusion Flame in a Boundary Layer; Analysis of a Boundary Layer with Large Density Gradient;

HEAT AND MASS TRANSFER

Transfer Based on Molecular Exchange

Heat Transfer

$$\dot{q}'' = -\lambda \nabla T$$



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$$\lambda\left\{\left(\frac{\partial^2 T}{\partial x^2}\right)_{x=x}+\left(\frac{\partial^2 T}{\partial y^2}\right)_{y=y}+\left(\frac{\partial^2 T}{\partial z^2}\right)_{z=z}\right\}\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}t$$

$$\rho c\{T(t+\mathrm{d}t)-T(t)\}\mathrm{d}x\mathrm{d}y\mathrm{d}z \equiv \rho c\left(\frac{\partial T}{\partial t}\right)\mathrm{d}t\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$\rho c\frac{\partial T}{\partial t}=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

$$\frac{\partial T}{\partial t}=\alpha\nabla^2 T$$

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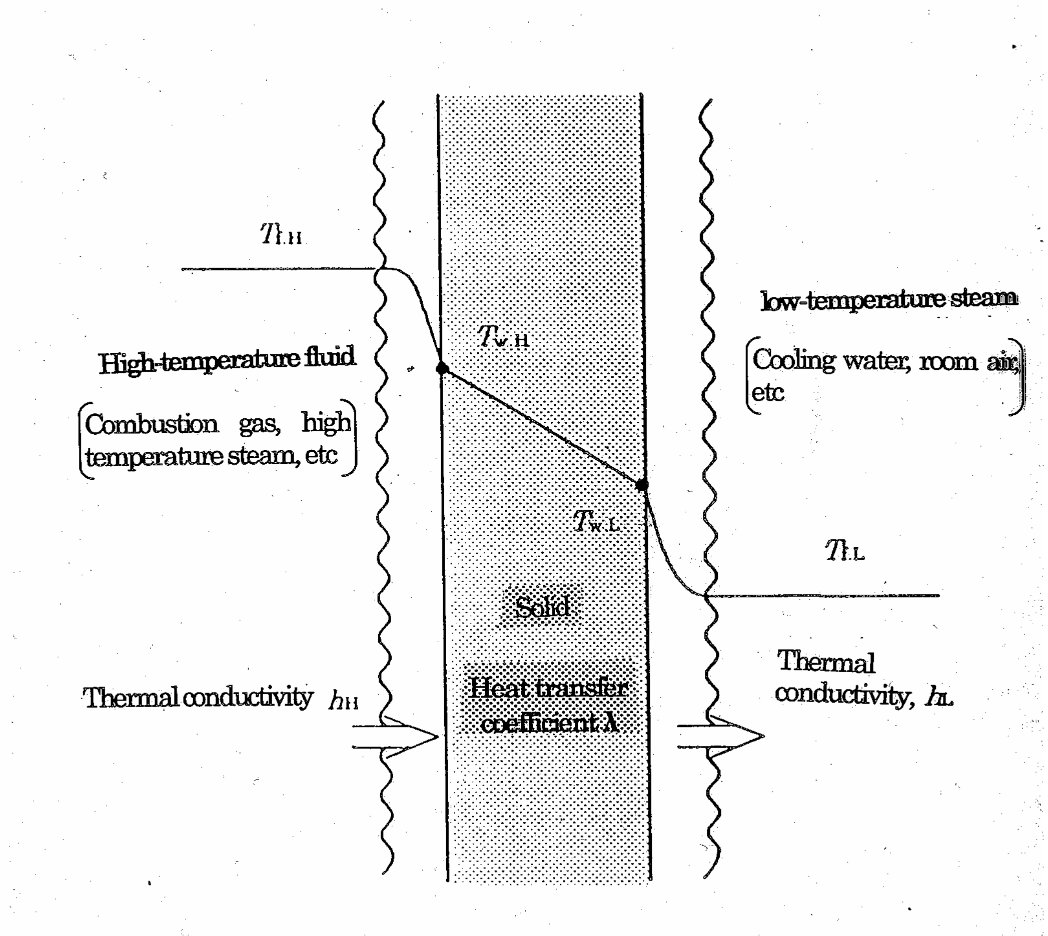

$$\dot{q}'' = h\left(T_f - T_w\right)$$

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Mass Transfer

$$\dot{m}''_{\mathrm{A}} = \rho_{\mathrm{A}}\left(v_{\mathrm{A}} - v\right) = -\rho D_{\mathrm{AB}} \nabla Y_{\mathrm{A}}$$

$$j^{*}_{\mathrm{A}} = C_{\mathrm{A}}\left(v^{*}_{\mathrm{A}} - v^{*}\right) = -c D_{\mathrm{AB}} \nabla X_{\mathrm{A}}$$

$$\dot{m}''_{i} = \rho_{A}\left(v_{A} - v\right) = -\rho D_{i} \nabla Y_{i}$$

$$\frac{\partial Y_i}{\partial t} = D_i \nabla^2 Y_i$$

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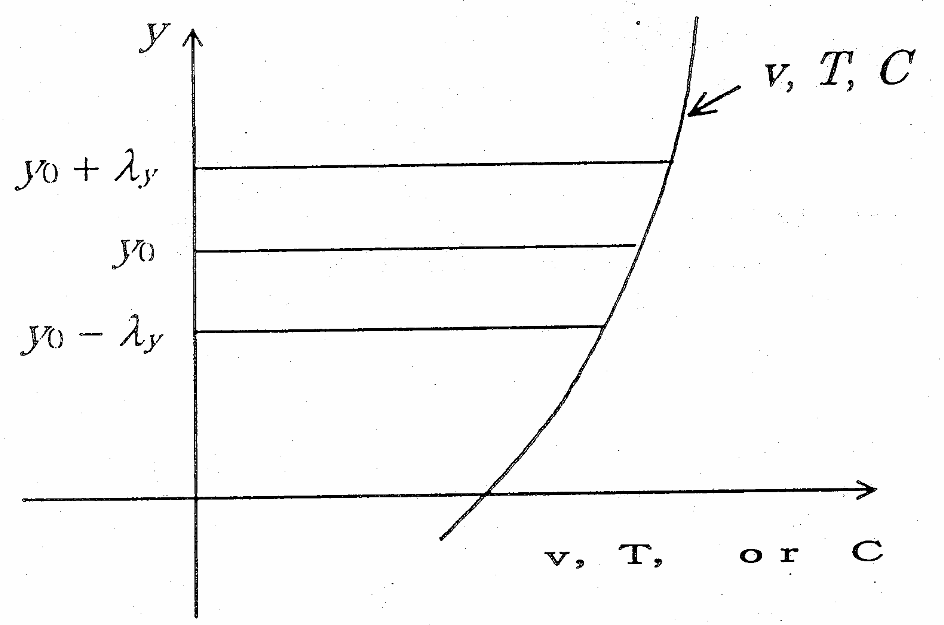
Explanation of Transfer Phenomena on the Basis of Gas Dynamics



$$N = \frac{1}{4} n\bar{c}$$

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$$G = N\{g(y_0 - \lambda_y) - g(y_0 + \lambda_y)\} = -\frac{1}{2} n\bar{c}\bar{\lambda}_y \left(\frac{\mathrm{d}g}{\mathrm{d}y}\right)_{y_0}$$

$$G = -\frac{1}{4} n\bar{c}\lambda_{\mathrm{m}} \left(\frac{\mathrm{d}ng}{\mathrm{d}y}\right)_{y_0}$$

$$\tau = -\frac{1}{4}\rho\bar{c}\lambda_{\mathrm{m}}\frac{\mathrm{d}v_x}{\mathrm{d}y} \qquad \tau = \mu\frac{\mathrm{d}v_x}{\mathrm{d}y} \qquad \nu = -\frac{1}{4}\bar{c}\lambda_{\mathrm{m}}$$

$$\dot{q}'' = -\frac{1}{4}\rho c_{\mathrm{p}}\bar{c}\lambda_{\mathrm{m}}\frac{\mathrm{d}T}{\mathrm{d}y} \qquad \dot{m}'' = -\frac{1}{4}\rho\bar{c}\lambda_{\mathrm{m}}\frac{\mathrm{d}Y_{\mathrm{i}}}{\mathrm{d}y}$$

$$\alpha = D_{\mathrm{i}} = \frac{1}{4}\bar{c}\lambda_{\mathrm{m}}$$

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Transfer Based on Turbulent Exchange

Characteristics of Turbulent Flow

$$u = \overline{u} + u'$$

$$\hat{u} = \sqrt{\overline{u'^2}}$$

$$c = \frac{\overline{u'(\tau)u'(\tau + t)}}{\{u'(\tau)\}^2}$$

$$f = \frac{\overline{u'(r)u'(r + x)}}{\{u'(r)\}^2}$$

$$g = \frac{\overline{u'(r)u'(r + y)}}{\{u'(r)\}^2}$$

$$g = 1 - \frac{y^2}{\lambda_g{}^2}$$

$$\Lambda_g = \int_0^\infty g\mathrm{d}y$$

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$$

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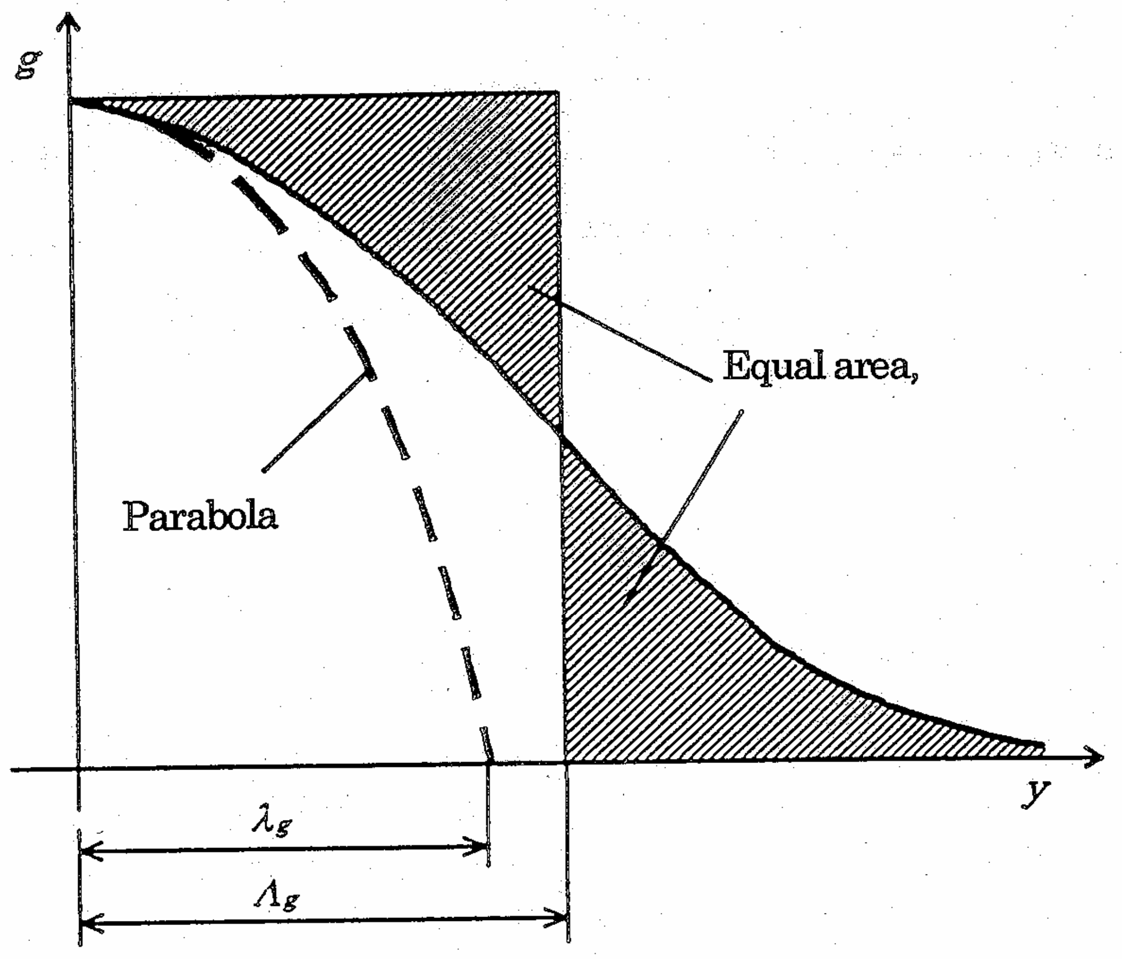

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Mixing Length Theory

$$\tau_1 = \mu \frac{\partial u}{\partial y} \qquad \tau_t = -\rho \overline{u'v'} = A_\tau \frac{\partial \bar{u}}{\partial y} \qquad \varepsilon_E = \frac{A_\tau}{\rho}$$

$$\overline{|v'|} = C_1 \overline{|u'|} = C_1 l \left| \frac{d\bar{u}}{dy} \right| \qquad \overline{u'v'} = -C_3 l^2 \left(\frac{d\bar{u}}{dy} \right)^2$$

$$\overline{u'v'} = -l^2 \left(\frac{d\bar{u}}{dy} \right)^2 \qquad \tau_t = -\rho \cdot l^2 \left(\frac{d\bar{u}}{dy} \right)^2 \qquad \tau_t = -\rho \cdot l^2 \left| \frac{d\bar{u}}{dy} \right| \frac{d\bar{u}}{dy}$$

$$A_\tau = -\rho \cdot l^2 \left| \frac{d\bar{u}}{dy} \right| \qquad \varepsilon_E = l^2 \left| \frac{d\bar{u}}{d\,y} \right|$$

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$k-\varepsilon$ (Two Equations) Model

$$\mu_{\mathrm{t}} = C_{\mu}\bar{\rho}\frac{k^2}{\varepsilon}$$

$$\bar{\rho}\cdot\bar{v}_y\frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{\mu_{\mathrm{t}}}{\sigma_{\mathrm{k}}}+\mu\right)\frac{\partial k}{\partial y}\right] - \bar{\rho}\overline{v''_x v''_y}\frac{\partial v_z}{\partial y} - \frac{\mu_{\mathrm{t}}}{\bar{\rho}^2}\frac{\partial\bar{\rho}}{\partial x}\frac{\partial\bar{p}}{\partial x} - \bar{\rho}\varepsilon$$

$$\bar{\rho}\cdot\bar{v}_y\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{\mu_{\mathrm{t}}}{\sigma_{\mathrm{k}}}+\mu\right)\frac{\partial \varepsilon}{\partial y}\right] - C_{\varepsilon 1}\frac{\varepsilon}{k}\left(\bar{\rho}\cdot\overline{v_x'' v_y''}\frac{\partial v_x}{\partial y} - \frac{\mu_{\mathrm{t}}}{\bar{\rho}^2}\frac{\partial\bar{\rho}}{\partial x}\frac{\partial\bar{p}}{\partial x}\right) - C_{\varepsilon 2}\bar{\rho}\frac{\varepsilon^2}{k}$$

$C_{\mu} = 0.09$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $\sigma_{\mathrm{k}} = 1.0$, $\sigma_{\varepsilon} = 1.30$, $\sigma_{\mathrm{t}} = 0.7$

Thermophoresis, Soret Effect, and Dufour Effect

Thermophoresis

$$F_{\mathrm{T}} = -\frac{32}{15} \cdot \frac{R^2}{\bar{c}} k_{\mathrm{g}} (\nabla T)_{\mathrm{x}} = -2\pi\mu\nu \frac{R^2}{\lambda_{\mathrm{m}}} \cdot \frac{(\nabla T)_{\mathrm{x}}}{T_0}$$

$$F_{\mathrm{T}} = -\frac{12\pi\mu\nu R C_{\mathrm{s}} \left(\dfrac{k_{\mathrm{g}}}{k_{\mathrm{p}}} + C_{\mathrm{t}} \dfrac{\lambda_{\mathrm{m}}}{R} \right) \dfrac{(\nabla T)_x}{T_0}}{\left(1 + 3 C_{\mathrm{m}} \dfrac{\lambda_{\mathrm{m}}}{R} \right) \left(1 + 2 \dfrac{k_{\mathrm{g}}}{k_{\mathrm{p}}} + 2 C_{\mathrm{t}} \dfrac{\lambda_{\mathrm{m}}}{R} \right)}$$

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Soret Effect

$$\Delta \dot{m}''_{\mathrm{Ti}} = -D_{\mathrm{i}}^{\mathrm{T}} \frac{\nabla T}{T} \qquad k_{\mathrm{T}}(T) = \frac{X_1 X_2}{\rho Y_1 Y_2} \frac{D_1^{\mathrm{T}}}{D_{12}} = -\frac{\Delta X_1}{\ln\left(\frac{T'}{T}\right)}$$

$$\overline{T} = \left(\frac{TT'}{T' - T}\right) \ln\left(\frac{T'}{T}\right)$$

Dufour Effect

$$\Delta \dot{q}''_{\mathrm{i}} = RT \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\frac{X_{\mathrm{j}} D_{\mathrm{ij}}}{W_{\mathrm{i}} D_{\mathrm{ij}}}\right) \left(\overline{V}_{\mathrm{i}} - \overline{V}_{\mathrm{j}}\right)$$

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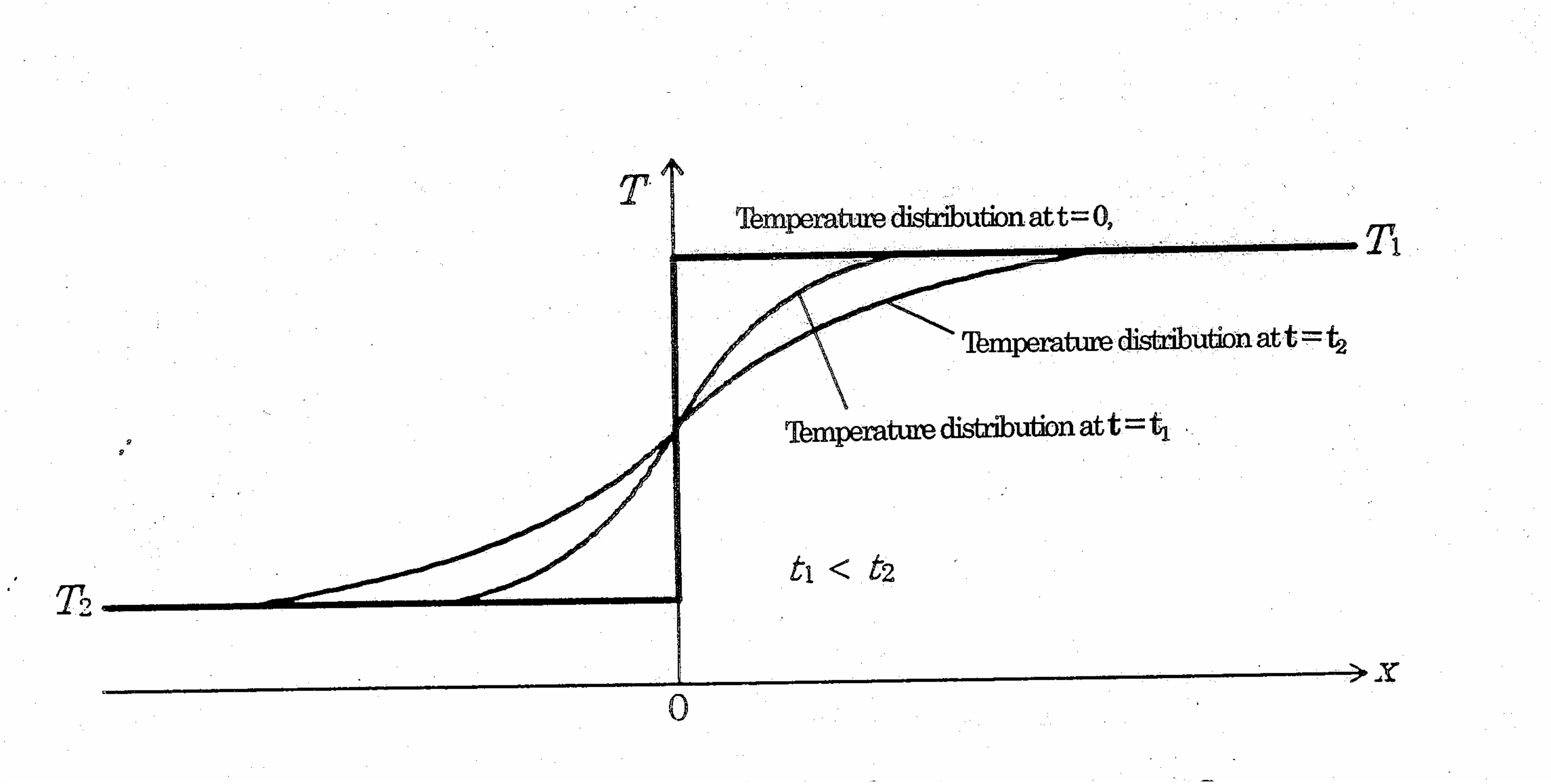
TEMPERATURE AND CONCENTRATION PROFILES

Variation of Temperature and Species Concentration in a Quiescent Atmosphere



$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

$$\left.\begin{array}{llll} t = 0: & T = T_1 & at & x \geq 0 \\ & T = T_2 & at & x < 0 \\ t > 0: & T = T_1 & for & x = \infty \\ & T = T_2 & for & x = -\infty \end{array}\right\}$$

$$\Theta = \frac{T - T_2}{T_1 - T_2}$$

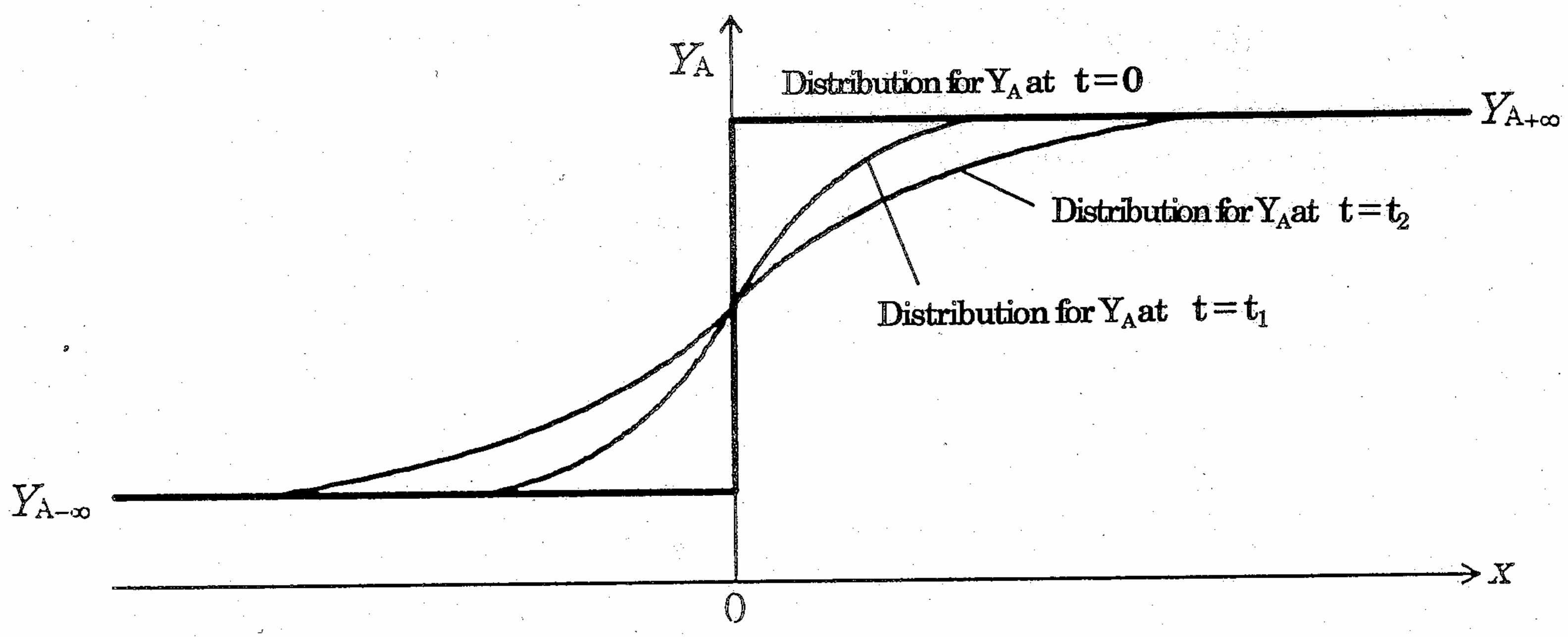
$$\left.\begin{array}{llll} t = 0: & \Theta = 1 & at & x \geq 0 \\ & \Theta = 0 & at & x < 0 \\ t > 0: & \Theta = 1 & for & x = \infty \\ & \Theta = 0 & for & x = -\infty \end{array}\right\}$$

$$\frac{\partial \Theta}{\partial t} = \alpha \frac{\partial^2 \Theta}{\partial x^2}$$

$$\Theta = \frac{1}{2}\left\{1 + \mathrm{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right\}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-\xi^2\right) \mathrm{d}\xi$$

Variation of Species Concentration Profile



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$$\frac{\partial Y_{\mathrm{A}}}{\partial t} = D_{\mathrm{A}} \nabla^2 Y_{\mathrm{A}}$$

$$\left.\begin{array}{lllll} t = 0: & Y_{\mathrm{A}} = Y_{\mathrm{A}+\infty} & at & x \geq 0 \\ & Y_{\mathrm{A}} = Y_{\mathrm{A}-\infty} & at & x < 0 \\ t > 0: & Y_{\mathrm{A}} = Y_{\mathrm{A}+\infty} & for & x = \infty \\ & Y_{\mathrm{A}} = Y_{\mathrm{A}-\infty} & for & x = -\infty \end{array}\right\}$$

$$\Phi = \frac{Y_{\mathrm{A}} - Y_{\mathrm{A}-\infty}}{Y_{\mathrm{A}+\infty} - Y_{\mathrm{A}-\infty}}$$

$$\frac{\partial \Phi}{\partial t} = D_{\mathrm{A}} \frac{\partial^2 \Phi}{\partial x^2}$$

$$\left.\begin{array}{lllll} t = 0: & \Phi = 1 & at & x \geq 0 \\ & \Phi = 0 & at & x < 0 \\ t > 0: & \Phi = 1 & for & x = \infty \\ & \Phi = 0 & for & x = -\infty \end{array}\right\}$$

$$\Phi = \frac{1}{2}\left\{1 + \mathrm{erf}\left(\frac{x}{2\sqrt{D_{\mathrm{A}} t}}\right)\right\}$$

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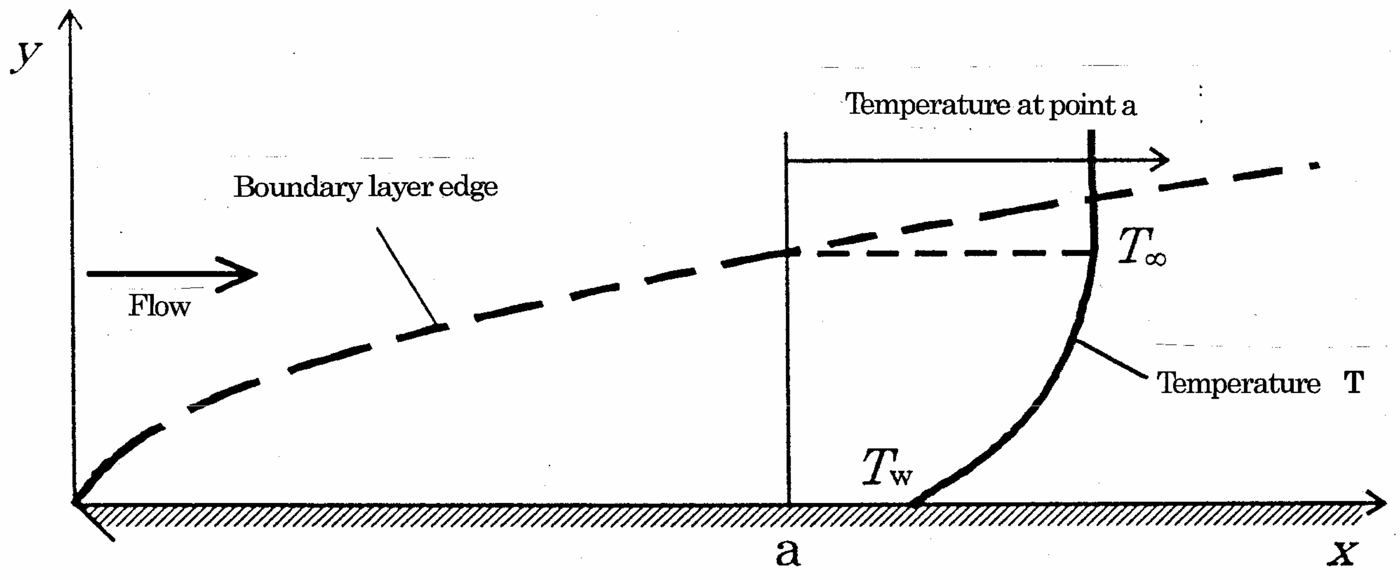
Variation of Temperature and Species Concentration in a Laminar Flow

Temperature and Species Concentration Profiles in the Boundary Layer



$$\frac{\mathrm{DT}}{\mathrm{D}t} = \alpha \nabla^2 T$$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$v_x = \frac{\partial \Psi}{\partial y}, \qquad v_y = -\frac{\partial \Psi}{\partial x} \qquad \Psi(x, y) = (\nu U x)^{\frac{1}{2}} f(\eta)$$

$$\eta \equiv y \left(\frac{U}{\nu x} \right)^{\frac{1}{2}} \qquad \Theta = \frac{T - T_{\mathrm{w}}}{T_\infty - T_{\mathrm{w}}}$$

$$2\Theta'' + \mathrm{Pr}\, f \Theta' = 0 \qquad \left. \begin{array}{l} \eta = 0: \quad \Theta = 0 \\ \eta = \infty: \quad \Theta = 1 \end{array} \right\}$$

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$$\Theta = \frac{\int_0^{\eta} \exp\left(-\frac{1}{2} \int_0^{\eta} (\Pr f) \mathrm{d}\eta \right) \mathrm{d}\eta}{\int_0^{\infty} \exp\left(-\frac{1}{2} \int_0^{\eta} (\Pr f) \mathrm{d}\eta \right) \mathrm{d}\eta}$$

$$2 f''' + f f'' = 0$$

$$\left. \begin{array}{ll} \eta = 0 : & f' = f = 0 \\ \eta = \infty : & f' = 1 \end{array} \right\}$$

$$\Theta = f'$$

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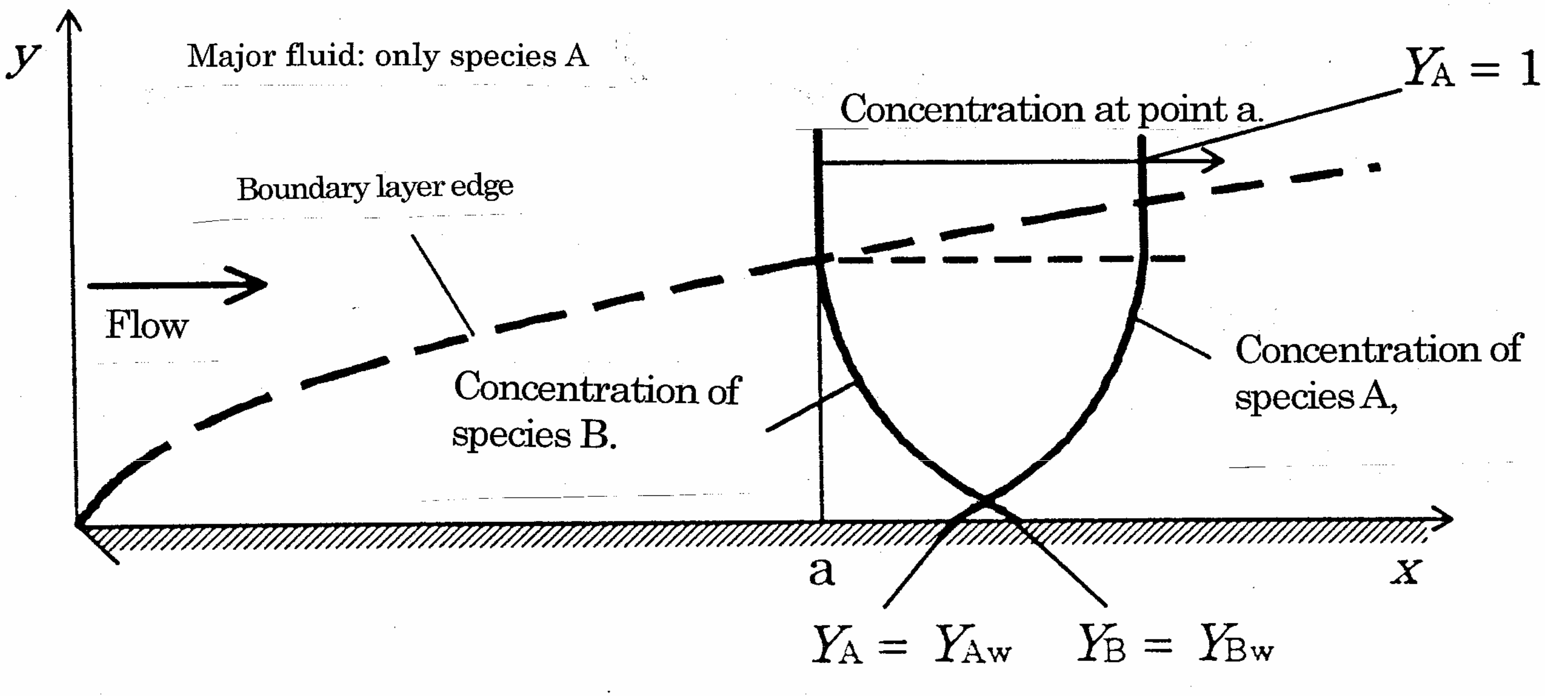

$$\frac{DY_A}{Dt} = D_A \nabla^2 Y_A$$

$$v_x \frac{\partial Y_A}{\partial x} + v_y \frac{\partial Y_A}{\partial y} = D_A \frac{\partial^2 Y_A}{\partial y^2}$$

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$$\Phi(\eta) = \frac{Y_{\mathrm{A}} - Y_{\mathrm{Aw}}}{1 - Y_{\mathrm{Aw}}}$$

$$2\Phi'' + \mathrm{Sc} f \Phi' = 0$$

$$\left.\begin{aligned} \eta = 0 : \quad & \Phi = 0 \\ \eta = \infty : \quad & \Phi = 1 \end{aligned}\right\}$$

$$\Phi = \frac{\int_0^{\eta} \exp\left(-\frac{1}{2}\int_0^{\eta}(\mathrm{Sc} f)\mathrm{d}\eta\right)\mathrm{d}\eta}{\int_0^{\infty} \exp\left(-\frac{1}{2}\int_0^{\eta}(\mathrm{Sc} f)\mathrm{d}\eta\right)\mathrm{d}\eta}$$

$$\Phi = f'$$

$$Y_{\mathrm{A}} = (1 - Y_{\mathrm{Aw}})\Phi + Y_{\mathrm{Aw}}$$

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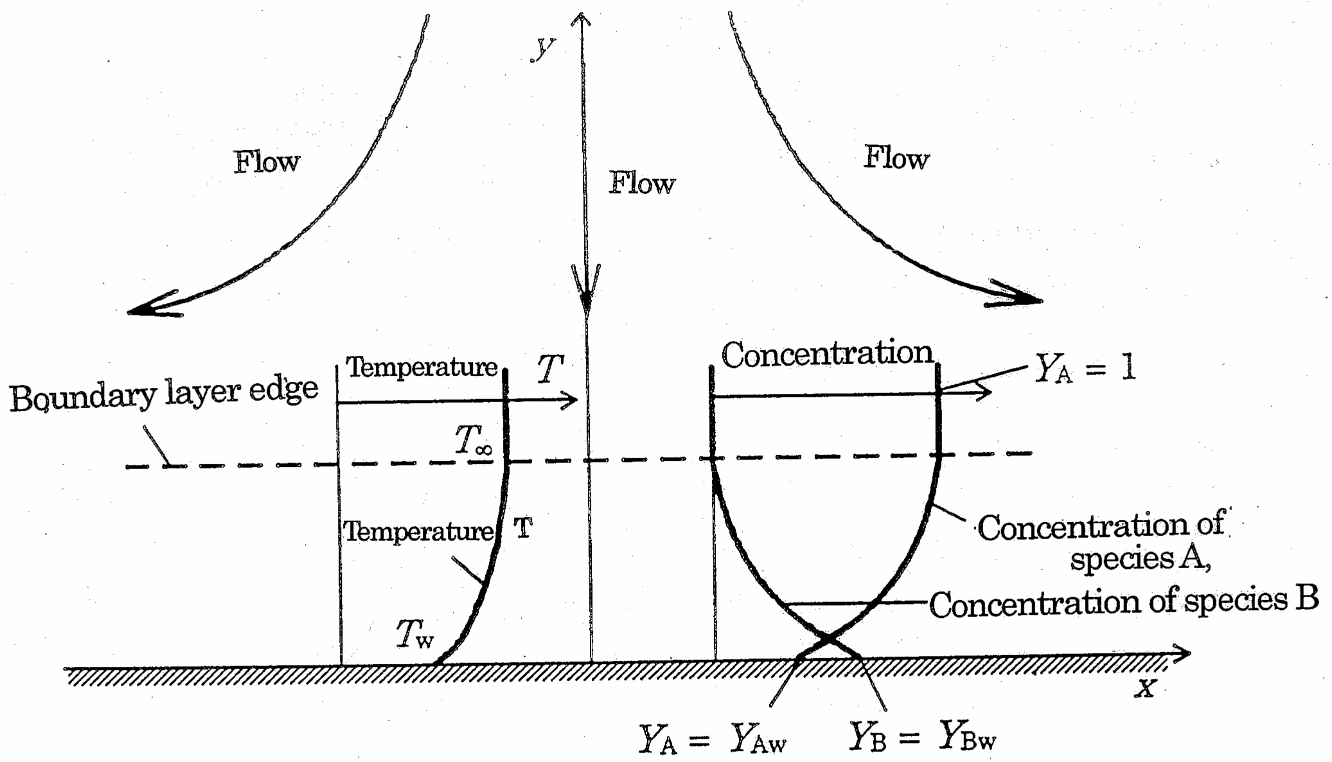

$$f(\eta) = \frac{\Psi(x,y)}{(\nu U_\infty x)^{\frac{1}{2}}} = \frac{\Psi(x,y)}{x(a\nu)^{\frac{1}{2}}}$$

$$\eta \equiv \left(\frac{a}{\nu}\right)^{\frac{1}{2}} y$$

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$$\Phi'' + \mathrm{Sc} f \Phi' = 0$$

$$\Theta'' + \mathrm{Pr}\, f \Theta' = 0$$

$$\left.\begin{aligned} \eta = 0 : \quad & \Phi = \Theta = 0 \\ \eta = \infty : \quad & \Phi = \Theta = 1 \end{aligned}\right\}$$

$$\Phi = \frac{\int_0^\eta \exp\left(-\int_0^\eta (\mathrm{Sc} f)\mathrm{d}\eta\right)\mathrm{d}\eta}{\int_0^\infty \exp\left(-\int_0^\eta (\mathrm{Sc} f)\mathrm{d}\eta\right)\mathrm{d}\eta}$$

$$\Theta = \frac{\int_0^\eta \exp\left(-\int_0^\eta (\mathrm{Pr}\, f)\mathrm{d}\eta\right)\mathrm{d}\eta}{\int_0^\infty \exp\left(-\int_0^\eta (\mathrm{Pr} f)\mathrm{d}\eta\right)\mathrm{d}\eta}$$

$$f''' + ff'' - (f')^2 + 1 = 0$$

$$\left.\begin{aligned} \eta = 0 : \quad & f' = f = 0 \\ \eta = \infty : \quad & f' = 1 \end{aligned}\right\}$$

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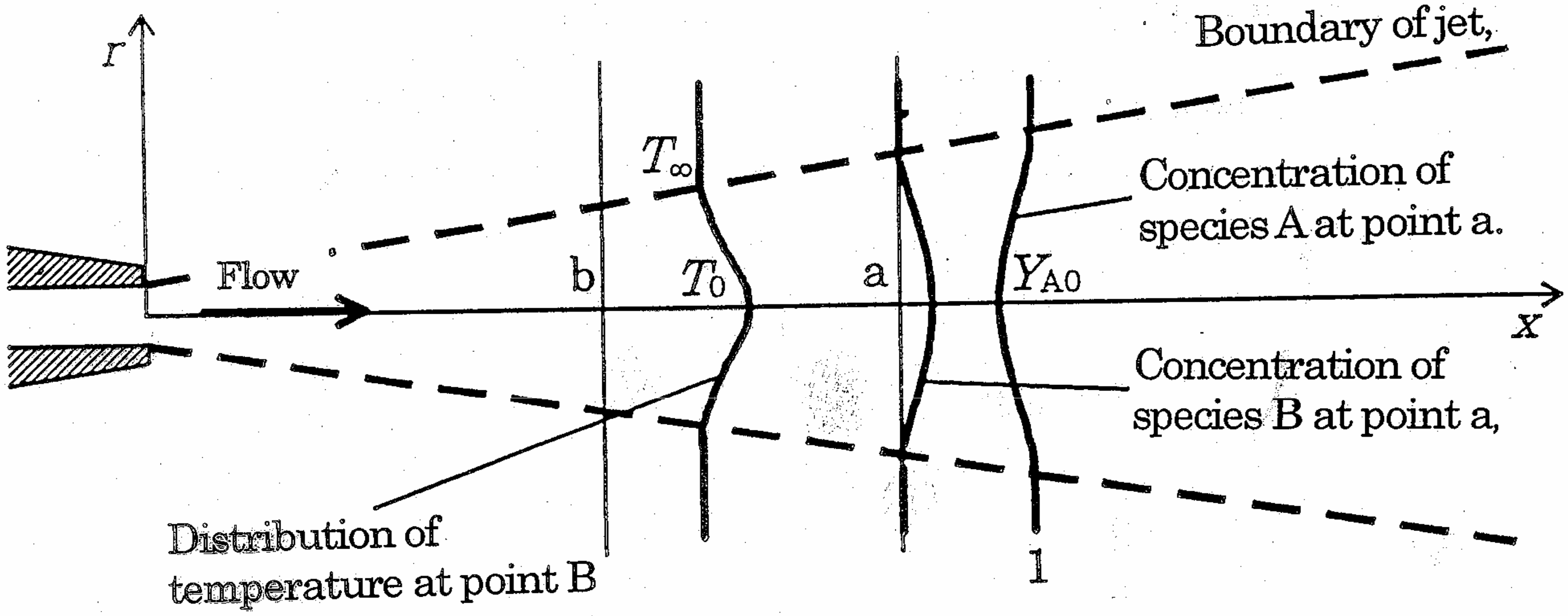
Temperature and Species Concentration Profiles in a Jet



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$$v_x \frac{\partial T}{\partial x} + v_r \frac{\partial T}{\partial r} = \alpha \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right\}$$

$$\eta \equiv \frac{r}{x} \qquad r v_x = \frac{\partial \Psi}{\partial r}, \qquad r v_r = -\frac{\partial \Psi}{\partial x}, \qquad \Psi = \nu x F(\eta)$$

$$\Theta = \frac{T_0 - T}{T_0 - T_{\mathrm{w}}}$$

$$\eta \Theta'' + \Theta' + \mathrm{Pr}\, F \Theta' = 0 \qquad \left. \begin{array}{ll} \eta = 0: & \Theta = 0 \\ \eta = \infty: & \Theta = 1 \end{array} \right\}$$

$$\Theta = \frac{\int_0^\eta \exp\left(-\int_0^\eta (\mathrm{Pr}\, F + 1) \mathrm{d}\eta \right) \mathrm{d}\eta}{\int_0^\infty \exp\left(-\int_0^\eta (\mathrm{Pr} F + 1) \mathrm{d}\eta \right) \mathrm{d}\eta}$$

$$\eta F'' + F F' - F' = 0 \qquad \eta = 0: \quad F(0) = F'(0) = 0$$

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$$v_x \frac{\partial Y_\mathrm{A}}{\partial x} + v_\mathrm{r} \frac{\partial Y_\mathrm{A}}{\partial r} = D_\mathrm{A} \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial Y_\mathrm{A}}{\partial r} \right) \right\}$$

$$\boldsymbol{\Phi} = \frac{Y_\mathrm{A} - Y_\mathrm{A0}}{1 - Y_\mathrm{A0}}$$

$$\eta \boldsymbol{\Phi}'' + \boldsymbol{\Phi}' + \mathrm{Sc} F \boldsymbol{\Phi}' = 0$$

$$\left. \begin{array}{ll} \eta = 0: & \boldsymbol{\Phi} = 0 \\ \eta = \infty: & \boldsymbol{\Phi} = 1 \end{array} \right\}$$

$$\boldsymbol{\Phi} = \frac{\int_0^\eta \exp\left(- \int_0^\eta (\mathrm{Sc} F + 1) \mathrm{d}\eta \right) \mathrm{d}\eta}{\int_0^\infty \exp\left(- \int_0^\eta (\mathrm{Sc} F + 1) \mathrm{d}\eta \right) \mathrm{d}\eta}$$

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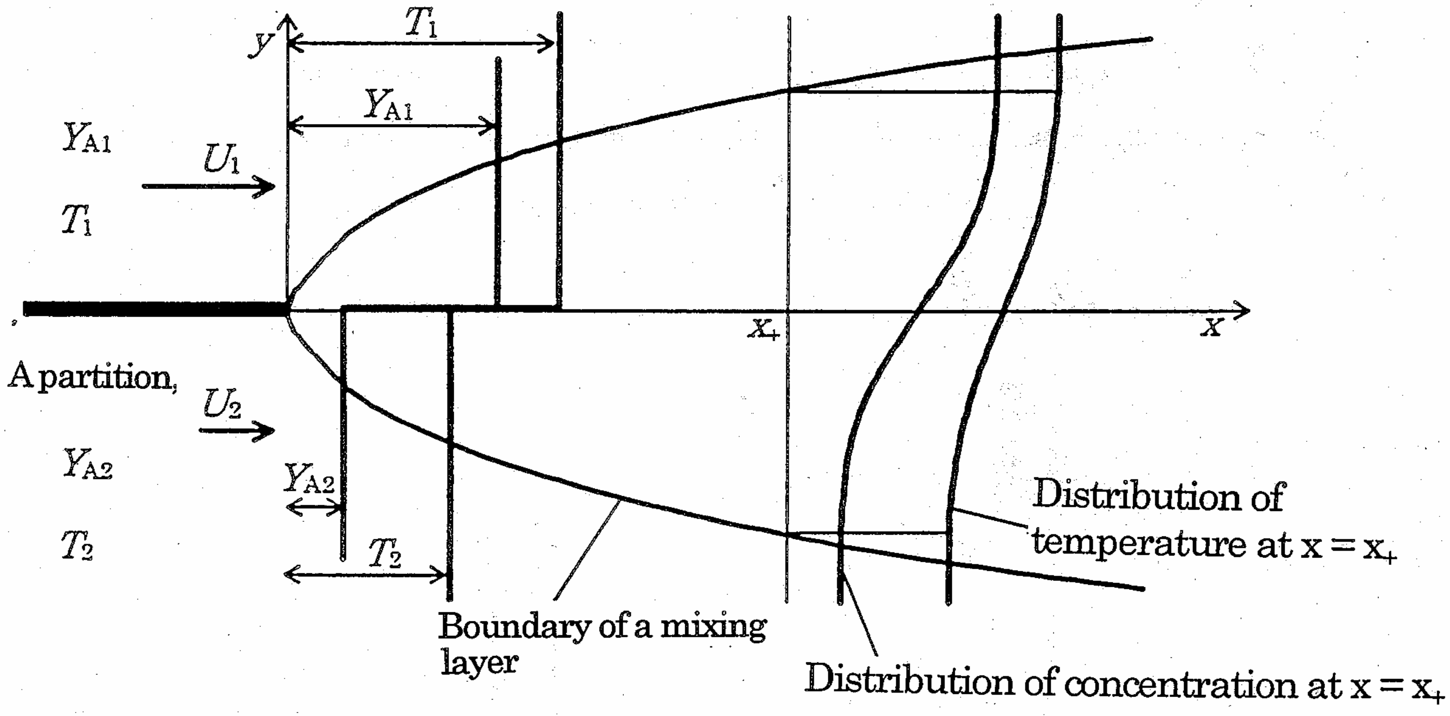
Temperature and Species Concentration Profiles in Parallel Flows of Different Velocities



$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$\eta = \left(\frac{U_1}{\nu x} \right)^{\frac{1}{2}} y \qquad \Psi = \left(\nu \cdot U_1 x \right)^{\frac{1}{2}} f \qquad \Theta = \frac{2T - T_1 - T_2}{T_1 - T_2}$$

$$2\Theta'' + \Pr f \Theta' = 0 \qquad \left. \begin{array}{l} \eta = -\infty : \quad \Theta = -1 \\ \eta = \infty : \quad \Theta = 1 \end{array} \right\}$$

$$\Theta = \frac{\int_0^\eta \exp\left(-\frac{1}{2} \int_0^\eta (\Pr f) \mathrm{d}\eta \right) \mathrm{d}\eta}{\int_0^\infty \exp\left(-\frac{1}{2} \int_0^\eta (\Pr f) \mathrm{d}\eta \right) \mathrm{d}\eta}$$

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$$v_x \frac{\partial Y_\mathrm{A}}{\partial x} + v_y \frac{\partial Y_\mathrm{A}}{\partial r} = D_\mathrm{A} \frac{\partial^2 Y_\mathrm{A}}{\partial y^2}$$

$$\boldsymbol{\Phi} = \frac{2Y_\mathrm{A} - Y_\mathrm{A2} - Y_\mathrm{A1}}{Y_\mathrm{A1} - Y_\mathrm{A2}}$$

$$2\boldsymbol{\Phi}'' + \mathrm{Sc} f \boldsymbol{\Phi}' = 0$$

$$\left.\begin{array}{l} \eta = -\infty : \quad \boldsymbol{\Phi} = -1 \\ \eta = \infty : \quad \boldsymbol{\Phi} = 1 \end{array}\right\}$$

$$\boldsymbol{\Phi} = \frac{\int_0^\eta \exp\left(-\frac{1}{2} \int_0^\eta (\mathrm{Sc} f) \mathrm{d}\eta \right) \mathrm{d}\eta}{\int_0^\infty \exp\left(-\frac{1}{2} \int_0^\eta (\mathrm{Sc}\, f) \mathrm{d}\eta \right) \mathrm{d}\eta}$$

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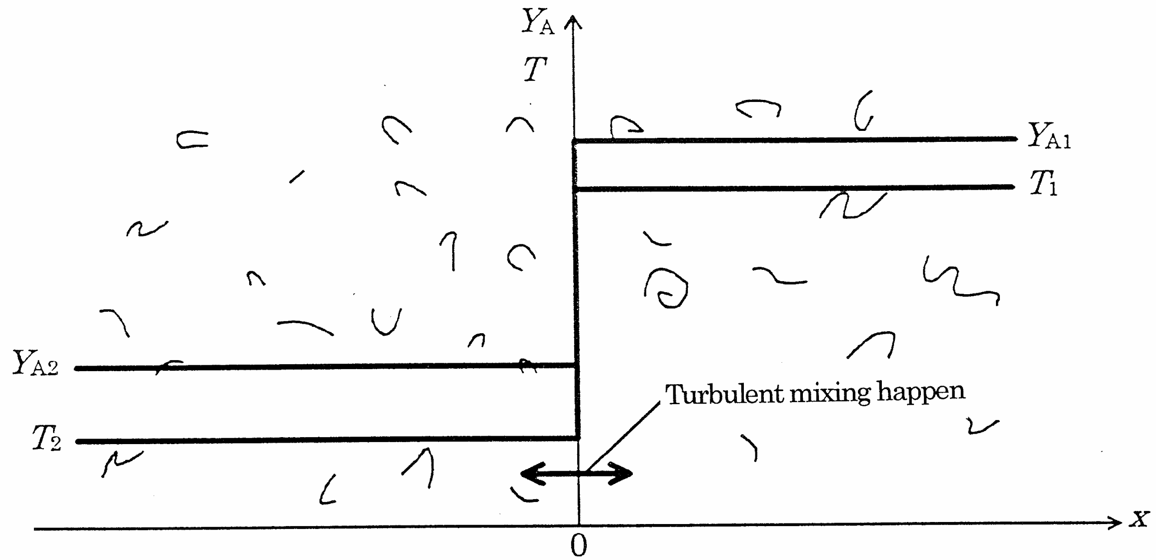
Temperature and Species Concentration Profiles in Turbulent Flows

Turbulent Diffusion at the Site with Temperature or Species Concentration Difference



$$t \le 0: \quad \left.\begin{array}{lll} \overline{T} = T_1 & \text{for} & x \ge 0 \\ \overline{T} = T_2 & \text{for} & x < 0 \end{array}\right\}$$

$$\frac{\mathrm{D}\overline{T}}{\mathrm{D}t} = \varepsilon_{\mathrm{T}} \nabla^2 \overline{T} \qquad \frac{\partial \overline{T}}{\partial t} = \varepsilon_{\mathrm{T}} \frac{\partial^2 \overline{T}}{\partial x^2}$$

$$\Theta = \frac{\overline{T} - T_2}{T_1 - T_2} \qquad \frac{\partial \Theta}{\partial t} = \varepsilon_{\mathrm{T}} \frac{\partial^2 \Theta}{\partial x^2}$$

$$\left.\begin{array}{llll} t = 0: & \Theta = 1 & \text{for} & x \ge 0 \\ & \Theta = 0 & \text{for} & x < 0 \\ t > 0: & \Theta = 1 & \text{for} & x = \infty \\ & \Theta = 0 & \text{for} & x = -\infty \end{array}\right\}$$

$$\Theta = \frac{1}{2}\left\{1 + \mathrm{erf}\left(\frac{x}{2\sqrt{\varepsilon_{\mathrm{T}} t}}\right)\right\}$$

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$$\varepsilon_{\mathrm{T}} = \lambda t \qquad \tau = t^2/2$$

$$\frac{\partial \Theta}{\partial \tau} = \lambda \frac{\partial^2 \Theta}{\partial x^2}$$

$$\left.\begin{array}{llll} \tau = 0: & \Theta = 1 & \text{for} & x \geq 0 \\ & \Theta = 0 & \text{for} & x < 0 \\ \tau > 0: & \Theta = 1 & \text{for} & x = \infty \\ & \Theta = 0 & \text{for} & x = -\infty \end{array}\right\}$$

$$\Theta = \frac{1}{2}\left\{1 + \mathrm{erf}\left(\frac{x}{2\sqrt{\lambda \tau}}\right)\right\} = \frac{1}{2}\left\{1 + \mathrm{erf}\left(\frac{x}{\sqrt{2\varepsilon_{\mathrm{T}} t}}\right)\right\}$$

The temperature profile change in the case
when the turbulent heat transfer coefficient increases with time

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$$t \le 0: \qquad \left.\begin{aligned} \overline{Y}_{\mathrm{A}} &= Y_{\mathrm{A1}} \quad \text{for} \quad x \ge 0 \\ \overline{Y}_{\mathrm{A}} &= Y_{\mathrm{A2}} \quad \text{for} \quad x < 0 \end{aligned}\right\}$$

$$\frac{\mathrm{D}\overline{Y}_{\mathrm{A}}}{\mathrm{D}t} = \varepsilon_{\mathrm{DA}} \nabla^2 \overline{Y}_{\mathrm{A}} \qquad \frac{\partial \overline{Y}_{\mathrm{A}}}{\partial t} = \varepsilon_{\mathrm{DA}} \frac{\partial^2 \overline{Y}_{\mathrm{A}}}{\partial x^2}$$

$$\Phi = \frac{\overline{Y}_{\mathrm{A}} - \overline{Y}_{\mathrm{A2}}}{\overline{Y}_{\mathrm{A1}} - \overline{Y}_{\mathrm{A2}}} \qquad \frac{\partial \Phi}{\partial t} = \varepsilon_{\mathrm{DA}} \frac{\partial^2 \Theta}{\partial x^2}$$

$$\left.\begin{aligned} t = 0: \quad & \Phi = 1 \quad \text{for} \quad x \ge 0 \\ & \Phi = 0 \quad \text{for} \quad x < 0 \\ t > 0: \quad & \Phi = 1 \quad \text{for} \quad x = \infty \\ & \Phi = 0 \quad \text{for} \quad x = -\infty \end{aligned}\right\}$$

$$\Phi = \frac{1}{2}\left\{1 + \operatorname{erf}\left(\frac{x}{2\sqrt{\varepsilon_{\mathrm{DA}} t}}\right)\right\}$$

$$\varepsilon_{\mathrm{DA}} = \kappa t \qquad \tau = t^2/2$$

$$\frac{\partial \Phi}{\partial \tau} = \kappa \frac{\partial^2 \Phi}{\partial x^2}$$

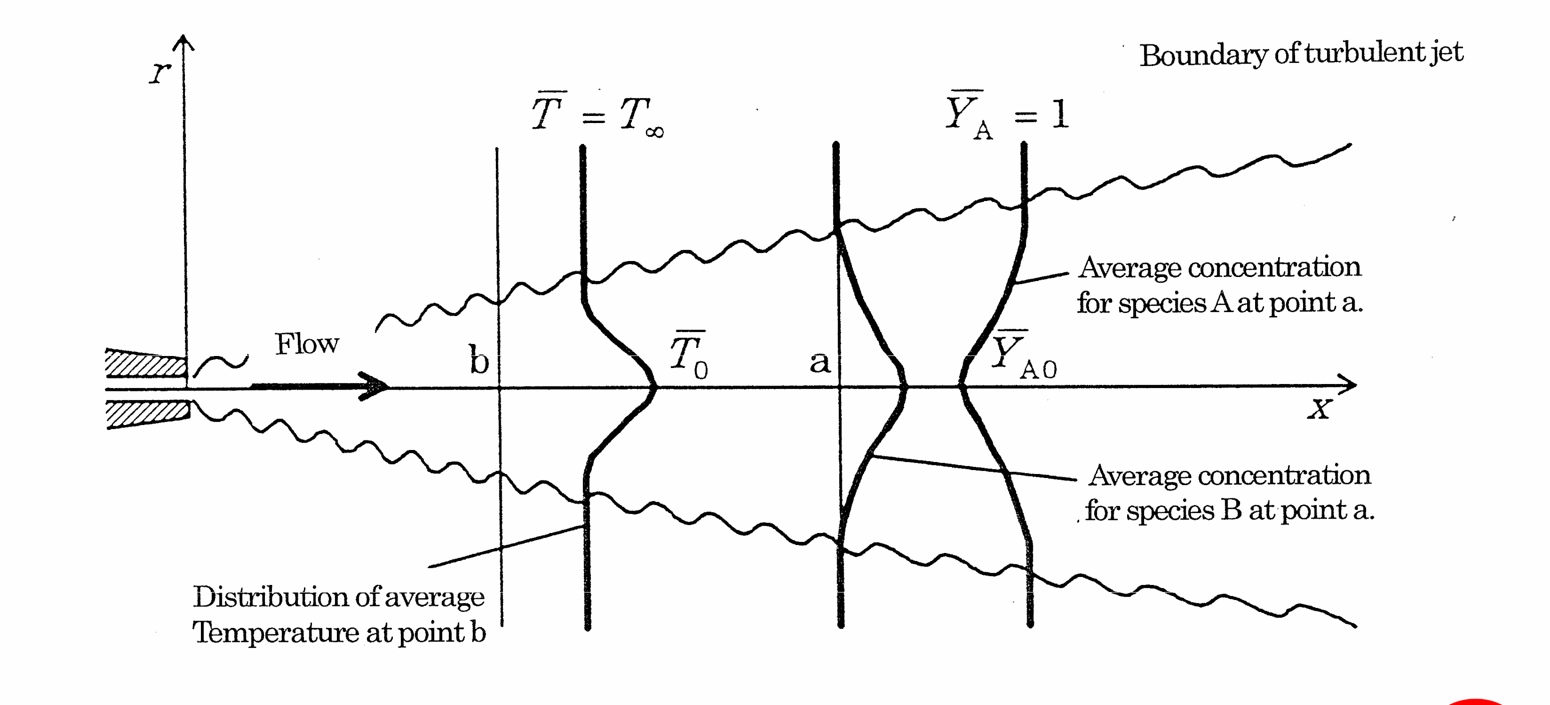
$$\left.\begin{array}{llll} \tau = 0: & \Phi = 1 & \text{for} & x \geq 0 \\ & \Phi = 0 & \text{for} & x < 0 \\ \tau > 0: & \Phi = 1 & \text{for} & x = \infty \\ & \Phi = 0 & \text{for} & x = -\infty \end{array}\right\}$$

$$\Phi = \frac{1}{2}\left\{1 + \mathrm{erf}\left(\frac{x}{2\sqrt{\kappa\tau}}\right)\right\} = \frac{1}{2}\left\{1 + \mathrm{erf}\left(\frac{x}{\sqrt{2\varepsilon_{\mathrm{DA}} t}}\right)\right\}$$

The species concentration profile change in the case
when the turbulent mass transfer coefficient increases with time

HIRANO

Temperature and Species Concentration Profiles across a Turbulent Jet



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$$v_x \frac{\partial \overline{T}}{\partial x} + v_r \frac{\partial \overline{T}}{\partial r} = \varepsilon_{\mathrm{T}} \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \overline{T}}{\partial r} \right) \right\}$$

$$\xi \equiv \frac{r}{x} \qquad r\overline{v}_x = \frac{\partial \Psi}{\partial r}, \qquad r\overline{v}_r = -\frac{\partial \Psi}{\partial x}, \qquad \Psi = \varepsilon_{\mathrm{T}} x F(\xi) \qquad \Theta = \frac{\overline{T}_0 - \overline{T}}{\overline{T}_0 - T_{\mathrm{w}}}$$

$$\xi \Theta'' + \Theta' + \mathrm{Pr_t}\, F \Theta' = 0 \qquad \left. \begin{array}{ll} \xi = 0: & \Theta = 0 \\ \xi = \infty: & \Theta = 1 \end{array} \right\}$$

$$\Theta = \frac{\int_0^{\xi} \exp\left(-\int_0^{\xi} (\mathrm{Pr_t}\, F + 1) \mathrm{d}\xi \right) \mathrm{d}\xi}{\int_0^{\infty} \exp\left(-\int_0^{\xi} (\mathrm{Pr_t} F + 1) \mathrm{d}\xi \right) \mathrm{d}\xi}$$

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$$\bar{v}_x \frac{\partial \overline{Y}_{\mathrm{A}}}{\partial x} + \bar{v}_r \frac{\partial \overline{Y}_{\mathrm{A}}}{\partial r} = \varepsilon_{\mathrm{DA}} \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \overline{Y}_{\mathrm{A}}}{\partial r} \right) \right\}$$

$$\boldsymbol{\Phi} = \frac{\overline{Y}_{\mathrm{A}} - \overline{Y}_{\mathrm{A0}}}{1 - \overline{Y}_{\mathrm{A0}}}$$

$$\xi \boldsymbol{\Phi}'' + \boldsymbol{\Phi}' + \mathrm{Sc}_{\mathrm{t}} F \boldsymbol{\Phi}' = 0$$

$$\left. \begin{array}{ll} \xi = 0 : & \boldsymbol{\Phi} = 0 \\ \xi = \infty : & \boldsymbol{\Phi} = 1 \end{array} \right\}$$

$$\boldsymbol{\Phi} = \frac{\int_0^{\xi} \exp\left(- \int_0^{\xi} (\mathrm{Sc}_{\mathrm{t}} F + 1) \mathrm{d}\xi \right) \mathrm{d}\xi}{\int_0^{\infty} \exp\left(- \int_0^{\xi} (\mathrm{Sc}_{\mathrm{t}} F + 1) \mathrm{d}\xi \right) \mathrm{d}\xi}$$

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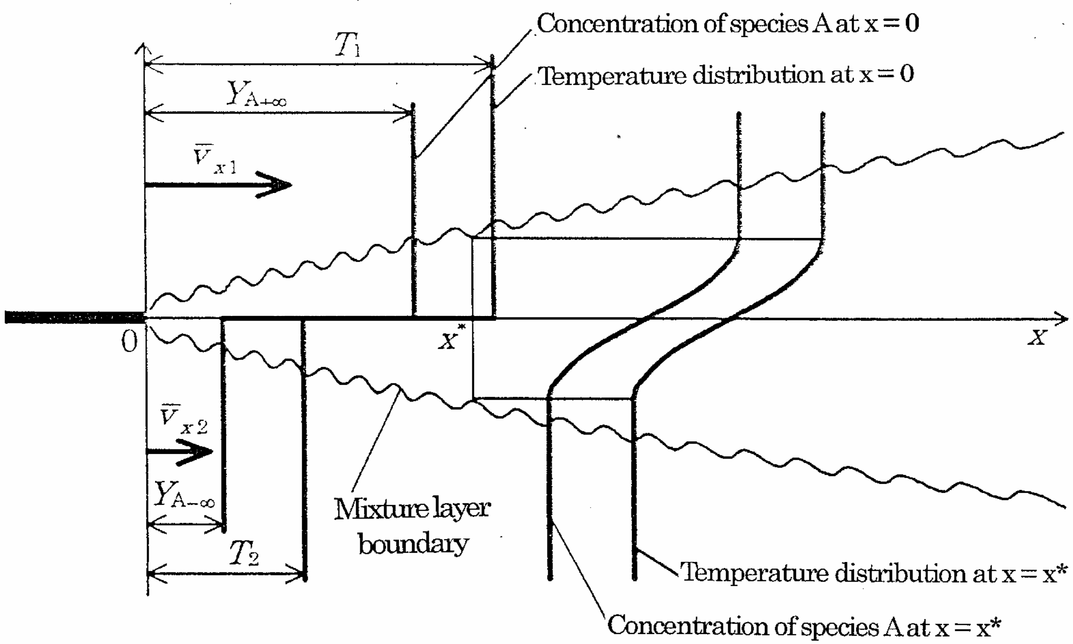
Temperature and Species Concentration Profiles across a Shear Flow



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$$\bar{v}_x \frac{\partial \bar{T}}{\partial x} + \bar{v}_y \frac{\partial \bar{T}}{\partial y} = \varepsilon_{\mathrm{T}} \frac{\partial^2 \bar{T}}{\partial y^2}$$

$$\left.\begin{aligned} &\Psi = xUF(\xi), \qquad \xi = \frac{\sigma y}{x}, \qquad \sigma = (4\lambda\kappa_1 c)^{-\frac{1}{3}}, \qquad \lambda = \frac{\bar{v}_{x1} - \bar{v}_{x2}}{\bar{v}_{x1} + \bar{v}_{x2}} \\ &\Theta = \frac{2\bar{T} - T_1 - T_2}{T_1 - T_2}, \qquad U = \frac{1}{2}(\bar{v}_{x1} - \bar{v}_{x2}), \qquad \mathrm{Pr_t} = \frac{\varepsilon_{\mathrm{E}}}{\varepsilon_{\mathrm{T}}} \end{aligned}\right\}$$

$$\Theta + 2\sigma^2 \,\mathrm{Pr_t}\, F\Theta' = 0$$

$$\left.\begin{aligned} &\xi = -\infty: \quad \Theta = -1 \\ &\xi = +\infty: \quad \Theta = 1 \end{aligned}\right\}$$

$$x\bar{v}_x / (2\varepsilon_{\mathrm{E}}) = 2\sigma^2 \,\mathrm{Pr_t},$$

$$\Theta = \frac{\int_0^{\xi} \exp\left(-2\sigma \int_0^{\xi} (\mathrm{Pr_t}\, F)\,\mathrm{d}\xi\right)\mathrm{d}\xi}{\int_0^{\infty} \exp\left(-2\sigma \int_0^{\xi} (\mathrm{Pr_t} F)\,\mathrm{d}\xi\right)\mathrm{d}\xi}$$

$$\Theta = F'$$

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$$\bar{v}_x \frac{\partial \bar{Y}_{\mathrm{A}}}{\partial x} + \bar{v}_y \frac{\partial \bar{Y}_{\mathrm{A}}}{\partial y} = \varepsilon_{\mathrm{DA}} \frac{\partial^2 \bar{Y}_{\mathrm{A}}}{\partial y^2}$$

$$\boldsymbol{\Phi} = \frac{2\bar{Y}_{\mathrm{A}} - Y_{\mathrm{A}+\infty} - Y_{\mathrm{A}-\infty}}{Y_{\mathrm{A}+\infty} - Y_{\mathrm{A}-\infty}}, \qquad \mathrm{Sc}_{\mathrm{t}} = \frac{\varepsilon_{\mathrm{E}}}{\varepsilon_{\mathrm{DA}}}$$

$$\boldsymbol{\Phi}'' + 2\sigma^2 \mathrm{Sc}_{\mathrm{t}} F \boldsymbol{\Phi}' = 0$$

$$\left.\begin{aligned} \xi = -\infty : \quad \boldsymbol{\Phi} = -1 \\ \xi = +\infty : \quad \boldsymbol{\Phi} = 1 \end{aligned}\right\}$$

$$x\bar{v}_x / (2\varepsilon_{\mathrm{E}}) = 2\sigma^2 Sc_{\mathrm{t}},$$

$$\boldsymbol{\Phi} = \frac{\int_0^{\xi} \exp\left(-2\sigma \int_0^{\xi} (\mathrm{Sc}_{\mathrm{t}} F) \mathrm{d}\xi\right) \mathrm{d}\xi}{\int_0^{+\infty} \exp\left(-2\sigma \int_0^{\xi} (\mathrm{Sc}_{\mathrm{t}} F) \mathrm{d}\xi\right) \mathrm{d}\xi}$$

$$\boldsymbol{\Phi} = F'$$

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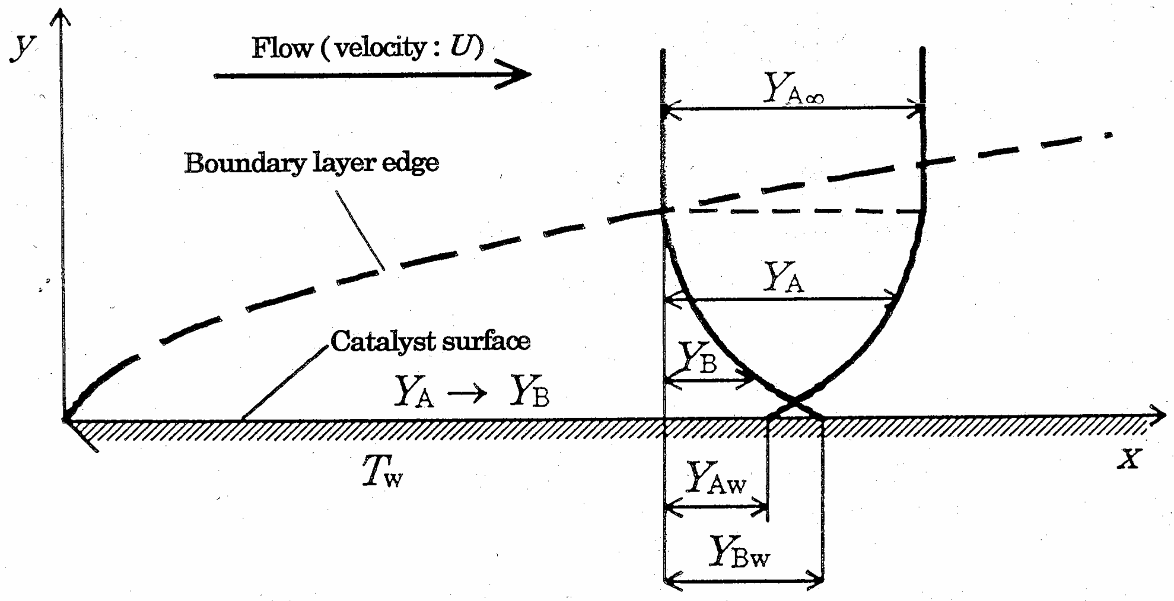
STRUCTURE OF REACTION SITES

Surface Reaction

Catalytic Reaction



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$$-\dot{m}''_{\mathrm{Aw}} = \rho_{\mathrm{w}} Y_{\mathrm{Aw}} B \exp\left(-\frac{E}{RT_{\mathrm{w}}}\right)$$

$$\rho_{\mathrm{w}} Y_{\mathrm{Aw}} B \exp\left(-\frac{E}{RT_{\mathrm{w}}}\right) = \rho_{\mathrm{w}} D_{\mathrm{A}} \left(\frac{\partial Y_{\mathrm{A}}}{\partial y}\right)_{\mathrm{w}}$$

$$\Phi = \frac{Y_{\mathrm{A}} - Y_{\mathrm{Aw}}}{Y_{\mathrm{A}\infty} - Y_{\mathrm{Aw}}}$$

$$Y_{\mathrm{Aw}} = \frac{D_{\mathrm{A}} Y_{\mathrm{A}\infty} \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} \Phi'_{\mathrm{w}}}{B \exp\left(-\frac{E}{RT_{\mathrm{w}}}\right) + D_{\mathrm{A}} \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} \Phi'_{\mathrm{w}}}$$

$$B \exp\left(-\frac{E}{RT_{\mathrm{w}}}\right) \ll D_{\mathrm{A}} \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} \Phi'_{\mathrm{w}}$$

$$Y_{\mathrm{Aw}} \cong Y_{\mathrm{A}\infty}$$

$$-\dot{m}''_{\mathrm{Aw}} = \rho_{\mathrm{w}} Y_{\mathrm{A}\infty} B \exp\left(-\frac{E}{RT_{\mathrm{w}}}\right)$$

$$B \exp\left(-\frac{E}{RT_{\mathrm{w}}}\right) \gg D_{\mathrm{A}} \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} \Phi'_{\mathrm{w}}$$

$$Y_{Aw} = \frac{D_{\mathrm{A}} Y_{\mathrm{A}\infty} \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} \Phi'_{\mathrm{w}}}{B \exp\left(-\frac{E}{RT_{\mathrm{w}}}\right)}$$

$$Y_{Aw} = \frac{D_{\mathrm{A}} Y_{\mathrm{A}\infty} \left(\frac{U}{\nu x} \right)^{\frac{1}{2}} \boldsymbol{\Phi}'_{\mathrm{w}}}{B \exp\left(- \frac{E}{R T_{\mathrm{w}}} \right)}$$

$$\frac{Y_{\mathrm{Aw}}}{Y_{\mathrm{A}\infty}} = \frac{D_{\mathrm{A}} \left(\frac{U}{\nu x} \right)^{\frac{1}{2}} \boldsymbol{\Phi}'_{\mathrm{w}}}{B \exp\left(- \frac{E}{R T_{\mathrm{w}}} \right)}$$

$$\eta \equiv \left(\frac{U}{\nu x} \right)^{\frac{1}{2}} y, \qquad \Psi(x, y) = (\nu U x)^{\frac{1}{2}} f(\eta)$$

$$- \dot{m}''_{\mathrm{Aw}} = \rho_{\mathrm{w}} D_{\mathrm{A}} \left(\frac{\partial Y_{\mathrm{A}}}{\partial y} \right)_{\mathrm{w}} = \rho_{\mathrm{w}} D_{\mathrm{A}} \left(Y_{\mathrm{A}\infty} - Y_{\mathrm{Aw}} \right) \left(\frac{U}{\nu x} \right)^{\frac{1}{2}} \boldsymbol{\Phi}'_{\mathrm{w}}$$

$$\cong \rho_{\mathrm{w}} D_{\mathrm{A}} \left(Y_{\mathrm{A}\infty} \right) \left(\frac{U}{\nu x} \right)^{\frac{1}{2}} \boldsymbol{\Phi}'_{\mathrm{w}}$$

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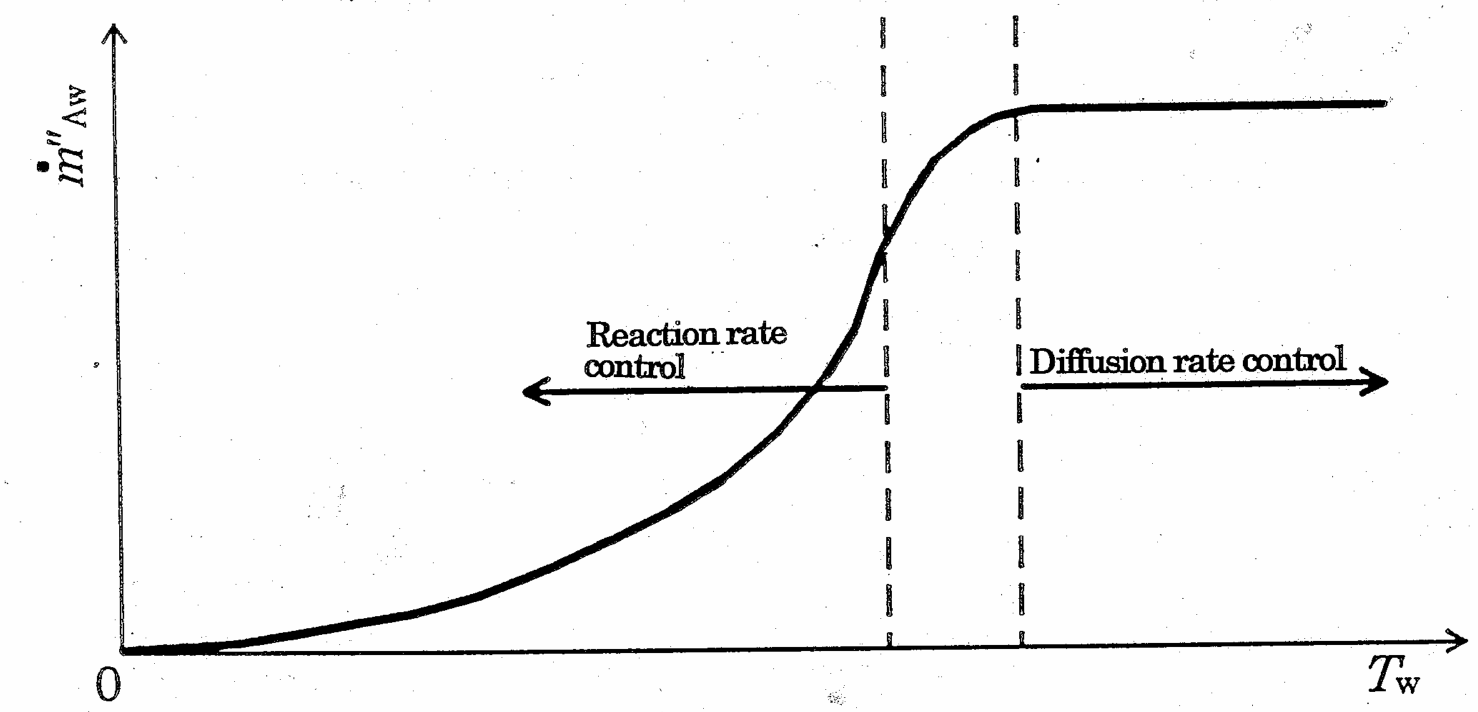

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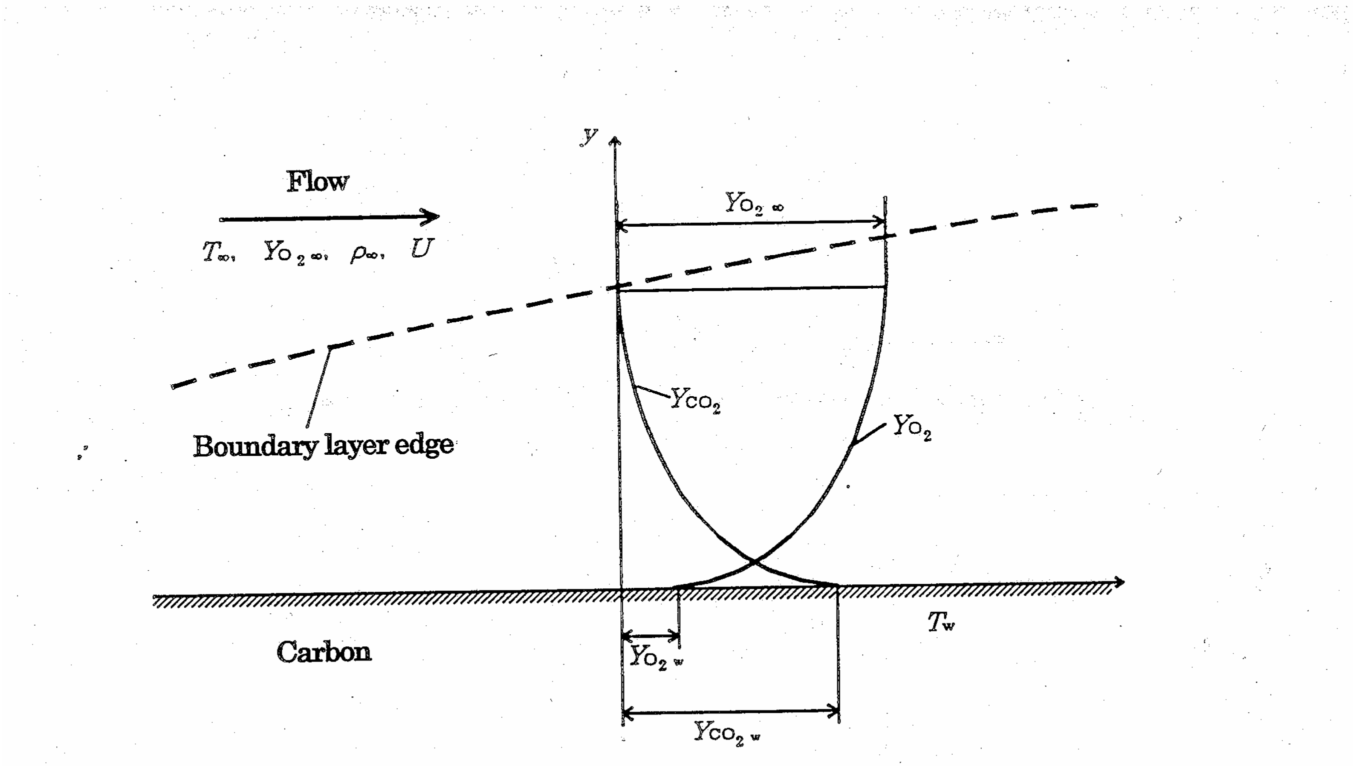
Combustion Reaction



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C(s) + O2=CO2
C(s) + CO2=2CO

$$\dot{m}''_{CO_2w} = \rho_w Y_{O_2w} B_1 \exp\left(-\frac{E_1}{RT_w}\right)$$

$$\dot{m}''_{COw} = \rho_w Y_{CO_2w} B_2 \exp\left(-\frac{E_2}{RT_w}\right)$$

$$\left(\dot{m}''_{CO_2w}\right)_{Net} = \dot{m}''_{CO_2w} - j\dot{m}''_{COw}$$

$$\rho_w Y_{O_2w} B_1 \exp\left(-\frac{E_1}{RT_w}\right) - j\rho_w Y_{CO_2w} B_2 \exp\left(-\frac{E_2}{RT_w}\right) = \rho_w D_{CO_2}\left(\frac{\partial Y_{CO_2}}{\partial y}\right)_w$$

$$\Phi_{CO_2} = \frac{Y_{CO_2w} - Y_{CO_2}}{Y_{CO_2w}}$$

$$Y_{CO_2w} = \frac{Y_{O_2w} B_1 \exp\left(-\frac{E_1}{RT_w}\right)}{jB_2 \exp\left(-\frac{E_2}{RT_w}\right) + D_{CO_2}\left(\frac{U}{\nu x}\right)^{\frac{1}{2}} \Phi'_{CO_2w}}$$

Low Temperature

$$\frac{Y_{CO_2w}}{Y_{O_2w}} = \frac{B_1 \exp\left(-\frac{E_{C1}}{RT_w}\right)}{D_{CO_2}\left(\frac{U}{\nu x}\right)^{\frac{1}{2}} \Phi'_{CO_2w}}$$

$$\left(\dot{m}''_{CO_2w}\right)_{Net} = \dot{m}''_{CO_2w}$$

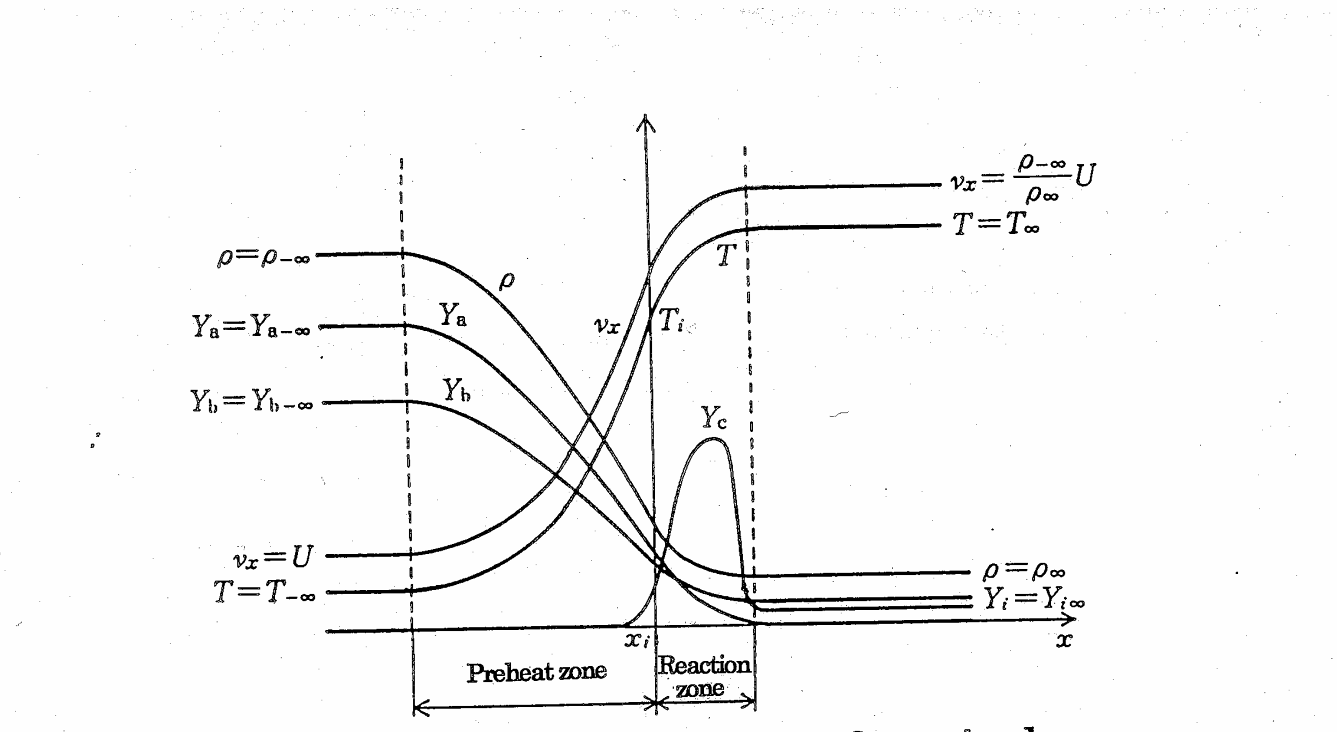
High Temperature

$$\frac{Y_{CO_2w}}{Y_{O_2w}} = \frac{B_1 \exp\left(-\frac{E_1}{RT_w}\right)}{jB_2 \exp\left(-\frac{E_2}{RT_w}\right)}$$

$$\left(\dot{m}''_{CO_2w}\right)_{Net} = \dot{m}''_{CO_2w} - j\dot{m}''_{COw} = 0$$

Reaction in a Gas Flow

Steady Premixed Flame



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$$\frac{\mathrm{d}(\rho v_x)}{\mathrm{d}x} = 0$$

$$\rho v_x \frac{\mathrm{d}v_x}{\mathrm{d}x} = -\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}x}\left(\mu \frac{\mathrm{d}v_x}{\mathrm{d}x}\right)$$

$$\rho c_p v_x \frac{\mathrm{d}T}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda \frac{\mathrm{d}T}{\mathrm{d}x}\right) + \dot{\omega}_{\mathrm{F}}''' \Delta H$$

$$\rho v_x \frac{\mathrm{d}Y_{\mathrm{i}}}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\rho D_{\mathrm{i}} \frac{\mathrm{d}Y_{\mathrm{i}}}{\mathrm{d}x}\right) + \dot{\omega}_{\mathrm{i}}'''$$

$$\dot{\omega}_{\mathrm{i}}''' = \sum_{\mathrm{j}} A_{\mathrm{ij}} B_{\mathrm{ij}} \exp\left(-\frac{E_{\mathrm{ij}}}{RT}\right)$$

$$pV = nRT$$

$$\left.\begin{array}{ll} x = -\infty: & v_x = U, \quad T = T_{-\infty}, \quad Y_{\mathrm{i}} = Y_{\mathrm{i}-\infty}, \quad \rho = \rho_{-\infty} \\ x = \infty: & v_x = \dfrac{\rho_{-\infty}}{\rho_{\infty}} U, \quad T = T_{\infty}, \quad Y_{\mathrm{i}} = Y_{\mathrm{i}\infty}, \quad \rho = \rho_{\infty} \end{array}\right\}$$

$$\rho_{-\infty} c_{\mathrm{p}} U \frac{\mathrm{d}T}{\mathrm{d}x} = \lambda \frac{\mathrm{d}^2 T}{\mathrm{d}x^2} + \dot{\omega}_{\mathrm{F}}''' \Delta H$$

$$\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{\mathrm{i}} = \frac{\rho c_{\mathrm{p}} U}{\lambda}\left(T_{\mathrm{i}} - T_{\mathrm{u}}\right)$$

$$\rho Y_{\mathrm{F}} U = -\dot{\omega}_{\mathrm{F}}''' \delta_{\mathrm{r}}$$

$$\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{\mathrm{i}} = \left(\frac{R}{\delta_{\mathrm{r}}}\right)\left(T_{\mathrm{b}} - T_{\mathrm{i}}\right)$$

$$\delta_{\mathrm{r}} = \left\{ R\left(\frac{T_{\mathrm{b}} - T_{\mathrm{i}}}{T_{\mathrm{i}} - T_{\mathrm{u}}}\right)\left(\frac{\lambda}{\rho c_{\mathrm{p}}}\right)\left(\frac{\rho Y_{\mathrm{F}}}{-\dot{\omega}_{\mathrm{F}}'''}\right)\right\}^{\frac{1}{2}}$$

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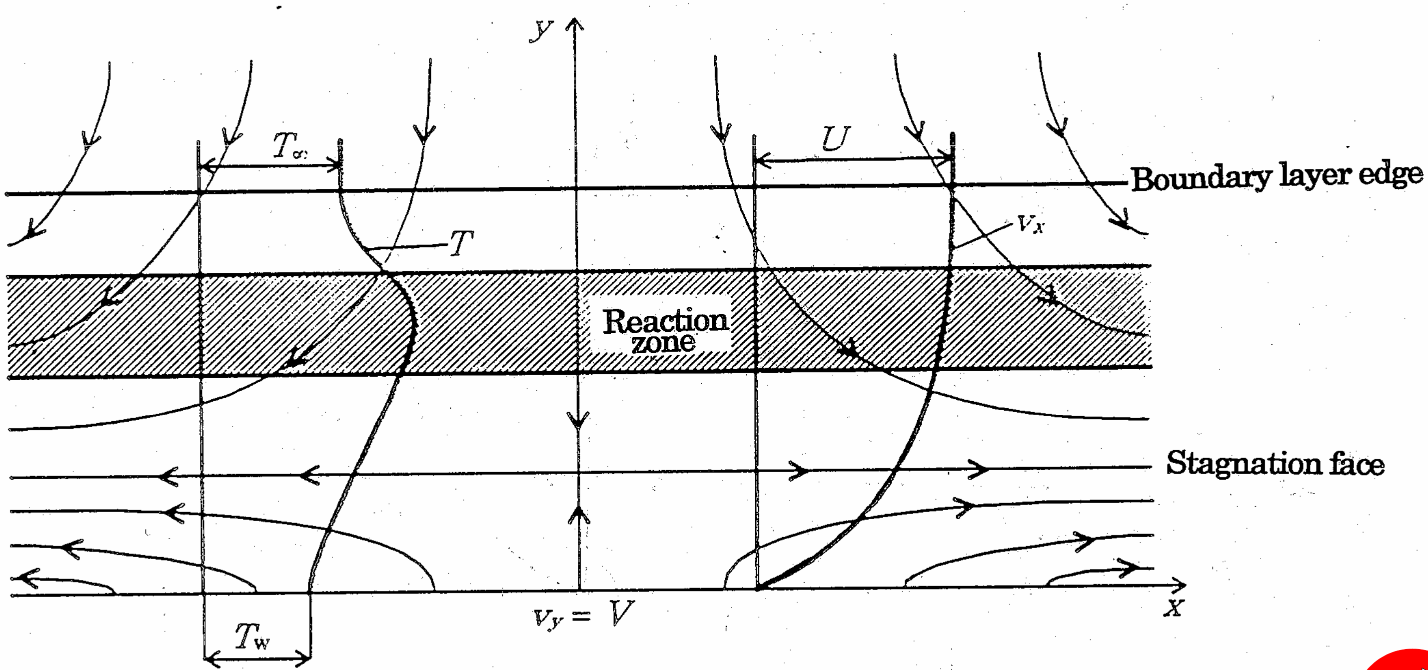
Diffusion Flame in a Boundary Layer



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$$\nu_{\mathrm{F}} W_{\mathrm{F}} + \nu_{\mathrm{O}} W_{\mathrm{O}} = \nu_{\mathrm{P}} W_{\mathrm{P}}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right)$$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \sum_{\mathrm{i}} \frac{\dot{\omega}_{\mathrm{i}}''' h^{(0)}}{\rho c_{\mathrm{p}}}$$

$$v_x \frac{\partial Y_{\mathrm{i}}}{\partial x} + v_y \frac{\partial Y_{\mathrm{i}}}{\partial y} = D_{\mathrm{i}} \frac{\partial^2 Y_{\mathrm{i}}}{\partial y^2} + \frac{\dot{\omega}_{\mathrm{i}}'''}{\rho} \quad (\mathrm{i} = \mathrm{F}, \mathrm{O}, \mathrm{P})$$

$$\dot{\omega}_F''' = -\nu_{\mathrm{F}} W_{\mathrm{F}} \left(\frac{\rho Y_{\mathrm{F}}}{W_{\mathrm{F}}}\right)^{\nu_{\mathrm{F}}} \left(\frac{\rho Y_{\mathrm{O}}}{W_{\mathrm{O}}}\right)^{\nu_{\mathrm{O}}} b \exp\left(-\frac{E}{RT}\right)$$

$$\dot{\omega}_{\mathrm{O}}''' = -\nu_{\mathrm{O}} W_{\mathrm{O}} \left(\frac{\rho Y_{\mathrm{F}}}{W_{\mathrm{F}}} \right)^{\nu_{\mathrm{F}}} \left(\frac{\rho Y_{\mathrm{O}}}{W_{\mathrm{O}}} \right)^{\nu_{\mathrm{O}}} b \exp\left(-\frac{E}{RT} \right)$$

$$\dot{\omega}_{\mathrm{P}}''' = -\nu_{\mathrm{P}} W_{\mathrm{P}} \left(\frac{\rho Y_{\mathrm{F}}}{W_{\mathrm{F}}} \right)^{\nu_{\mathrm{F}}} \left(\frac{\rho Y_{\mathrm{O}}}{W_{\mathrm{O}}} \right)^{\nu_{\mathrm{O}}} b \exp\left(-\frac{E}{RT} \right)$$

$$\sum_{\mathrm{i}} Y_{\mathrm{i}} = 1$$

$$\left. \begin{array}{l} y = 0: \quad v_x = 0, \quad v_y = V, \quad T = T_{\mathrm{w}}, \quad Y_{\mathrm{F}} = Y_{\mathrm{Fw}}, \quad Y_{\mathrm{N}} = Y_{\mathrm{Nw}} \\ y = \infty: \quad v_x = U, \quad T = T_{\infty}, \quad Y_{\mathrm{O}} = Y_{\mathrm{O}\infty}, \quad Y_{\mathrm{N}} = Y_{\mathrm{N}\infty} \end{array} \right\}$$

$$\Theta = \frac{T - T_{\mathrm{w}}}{T_{\infty} - T_{\mathrm{w}}} \qquad U = ax, \quad \frac{\mathrm{d}p}{\mathrm{d}x} = -\rho a^2 x, \quad \eta \equiv \left(\frac{a}{\nu} \right)^{\frac{1}{2}}, \quad f(\eta) = \frac{\Psi(x, y)}{x(a\nu)^{\frac{1}{2}}}$$

$$\Theta'' + \mathrm{Pr}\, f \Theta' - \frac{\mathrm{Pr}}{a(T_{\infty} - T_{\mathrm{w}})} \left(\sum_{\mathrm{i}} \frac{\dot{\omega}_{\mathrm{i}}''' h_{\mathrm{i}}^{(0)}}{\rho c_{\mathrm{p}}} \right) = 0 \qquad \left. \begin{array}{l} \eta = 0: \quad \Theta = 0 \\ \eta = \infty: \quad \Theta = 1 \end{array} \right\}$$

$$\Phi_{\mathrm{F}} = \frac{Y_{\mathrm{F}}}{Y_{\mathrm{Fw}}}, \quad \Phi_{\mathrm{O}} = \frac{Y_{\mathrm{O}}}{Y_{\mathrm{Ow}}}, \quad \Phi_{\mathrm{N}} = \frac{Y_{\mathrm{N}} - Y_{\mathrm{Nw}}}{Y_{\mathrm{N}\infty} - Y_{\mathrm{Nw}}}$$

$$\Phi'' + \mathrm{Sc}_{\mathrm{i}} f \Phi'_{\mathrm{i}} + \mathrm{Sc}_{\mathrm{i}} \frac{W_{\mathrm{i}}}{a\left(Y_{\mathrm{i}\infty} - Y_{\mathrm{iw}}\right)} \frac{\dot{\omega}'''_{\mathrm{i}}}{\rho} = 0 \quad \left(\mathrm{i} = \mathrm{F}, \mathrm{O}, \mathrm{P}\right)$$

$$\left.\begin{aligned} \eta = 0: &\quad \Phi_{\mathrm{F}} = 1, \quad \Phi_{\mathrm{O}} = 0, \quad \Phi_{\mathrm{N}} = 0 \\ \eta = \infty: &\quad \Phi_{\mathrm{F}} = 0, \quad \Phi_{\mathrm{O}} = 1, \quad \Phi_{\mathrm{N}} = 1 \end{aligned}\right\},$$

$$V = Y_{\mathrm{F}} V - D_{\mathrm{F}} \left(\frac{\partial Y_{\mathrm{F}}}{\partial y}\right)_{\mathrm{w}} \qquad 0 = Y_{\mathrm{Nw}} V - D_{\mathrm{N}} \left(\frac{\partial Y_{\mathrm{N}}}{\partial y}\right)_{\mathrm{w}} \qquad Y_{\mathrm{C}} \equiv \frac{Y_{\mathrm{F}}}{\nu_{\mathrm{F}} W_{\mathrm{F}}} - \frac{Y_{\mathrm{O}}}{\nu_{\mathrm{O}} W_{\mathrm{O}}}$$

$$v_x \frac{\partial Y_{\mathrm{C}}}{\partial x} + v_y \frac{\partial Y_{\mathrm{C}}}{\partial y} = D \frac{\partial^2 Y_{\mathrm{C}}}{\partial y^2} \qquad \Phi_{\mathrm{C}} = \frac{Y_{\mathrm{C}} - Y_{\mathrm{Cw}}}{Y_{\mathrm{C}\infty} - Y_{\mathrm{Cw}}}$$

$$\Phi''_{\mathrm{C}} + \mathrm{Sc}_{\mathrm{C}} f \Phi_{\mathrm{C}} \qquad \left.\begin{aligned} \eta = 0: &\quad \Phi_{\mathrm{C}} = 0 \\ \eta = \infty: &\quad \Phi_{\mathrm{C}} = 1 \end{aligned}\right\}$$

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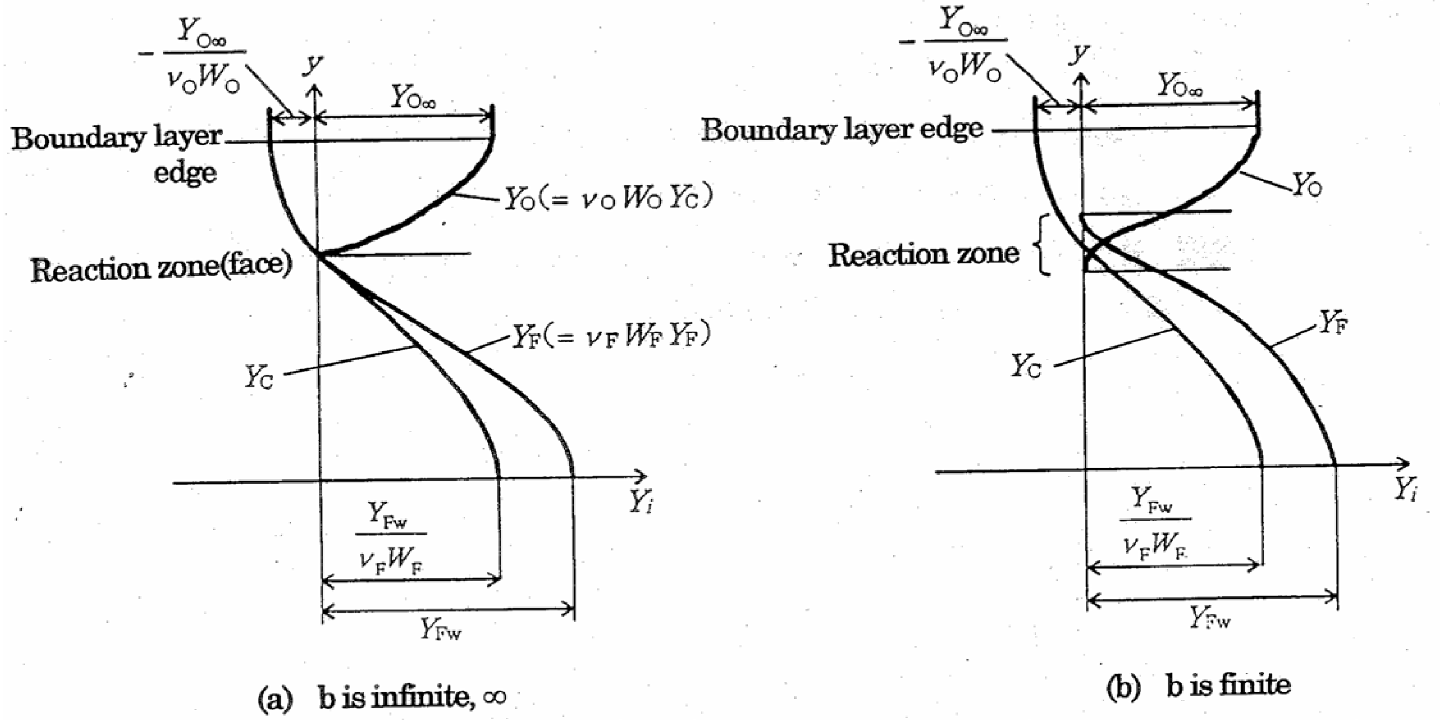

$$Y_C = \frac{Y_F}{\nu_F W_F} \qquad Y_C = -\frac{Y_O}{\nu_O W_O} \qquad \Phi_C = \frac{-Y_{Cw}}{Y_{C\infty} - Y_{Cw}}$$

CIS

$$v_x \frac{\partial Y_F}{\partial x} + v_y \frac{\partial Y_F}{\partial y} = D_F \frac{\partial^2 Y_F}{\partial y^2}$$

$$\left. \begin{array}{ll} y = 0: & V = Y_F V - D_F \dfrac{\partial^2 Y_F}{\partial y^2} \\ y = y^*: & Y_F = 0 \end{array} \right\}$$

$$v_x \frac{\partial Y_O}{\partial x} + v_y \frac{\partial Y_O}{\partial y} = D_O \frac{\partial^2 Y_O}{\partial y^2}$$

$$\left. \begin{array}{ll} y = y^*: & Y_O = 0 \\ y = \infty: & Y_O = Y_{O\infty} \end{array} \right\}$$

$$y = y^*: \quad D_F \frac{\partial Y_F}{\partial y} = \left(\frac{\nu_F W_F}{\nu_O W_O} \right) D_O \frac{\partial Y_O}{\partial y}$$

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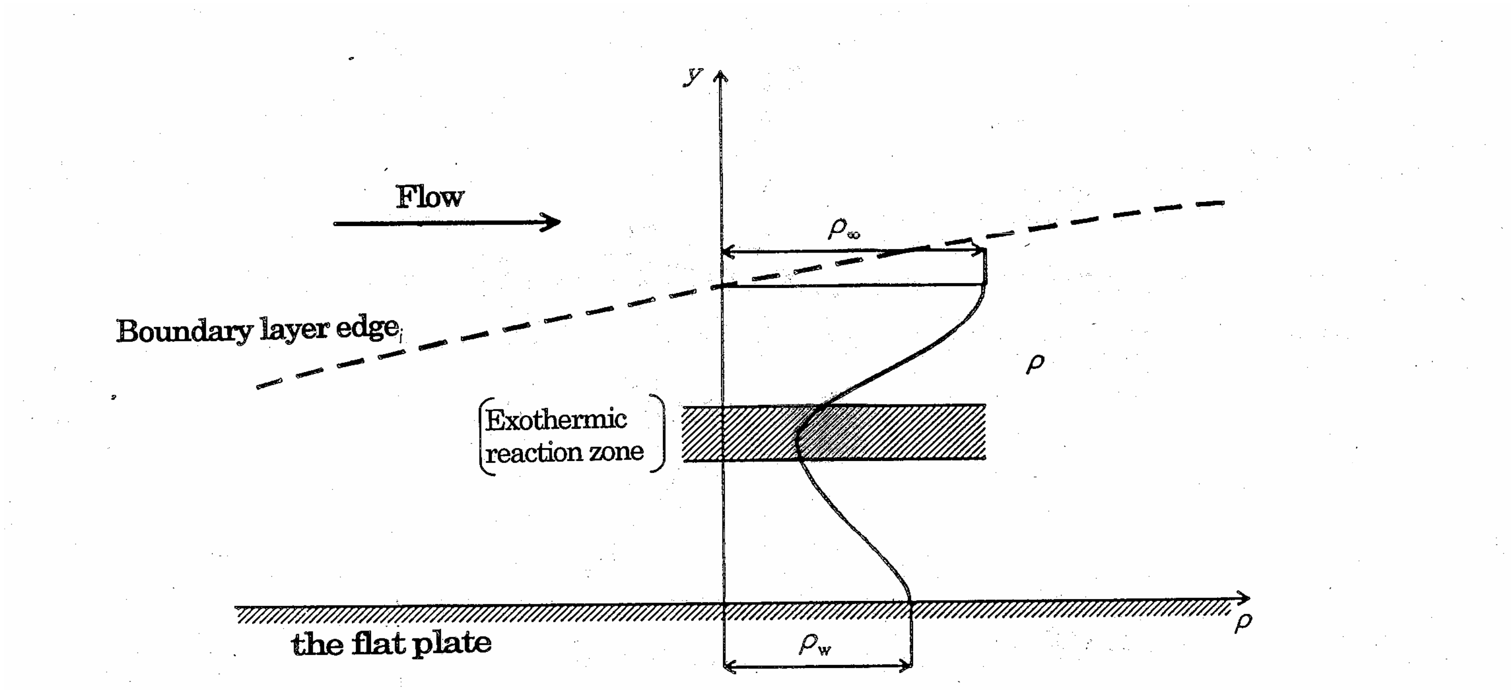
Analysis of a Boundary Layer with Large Density Gradient



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$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} = 0$$

$$\rho v_x \frac{\partial v_x}{\partial x} + \rho v_y \frac{\partial v_x}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial v_x}{\partial y}\right)$$

$$\rho c_\mathrm{p} v_x \frac{\partial T}{\partial x} + \rho c_\mathrm{p} v_y \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \dot{\omega}_\mathrm{F}''' \Delta H$$

$$\rho v_x \frac{\partial Y_\mathrm{i}}{\partial x} + \rho v_y \frac{\partial Y_\mathrm{i}}{\partial y} = \frac{\partial}{\partial y}\left(\rho D_\mathrm{i} \frac{\partial Y_\mathrm{i}}{\partial y}\right) + W_\mathrm{i} \dot{\omega}'''$$

$$\left.\begin{array}{ll} y = 0: & v_x = 0, \quad v_y = V, \quad T = T_\mathrm{w} \quad Y_\mathrm{i} = Y_\mathrm{iw} \\ y = \infty: & v_x = U, \quad T = T_\infty, \quad Y_\mathrm{i} = Y_{i\infty} \end{array}\right\}$$

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$$\left.\begin{aligned}&\xi = U\rho_\infty \mu_\infty x, \quad \eta = \left(\frac{U}{\nu_\infty x}\right)^{\frac{1}{2}} \int_0^y \frac{\rho}{\rho_\infty} \mathrm{d}y, \\ &\Psi = (U\rho_\infty \mu_\infty x) f(\eta), \quad \rho v_x = \frac{\partial \Psi}{\partial y}, \quad \rho v_y = -\frac{\partial \Psi}{\partial x}\end{aligned}\right\}$$

$$\rho\mu = \text{constant}$$

$$2f''' + ff'' = 0$$

$$\left.\begin{aligned}&\eta = 0: \quad f' = 0, \quad f = 0 \\ &\eta = \infty: \quad f' = 1\end{aligned}\right\}$$

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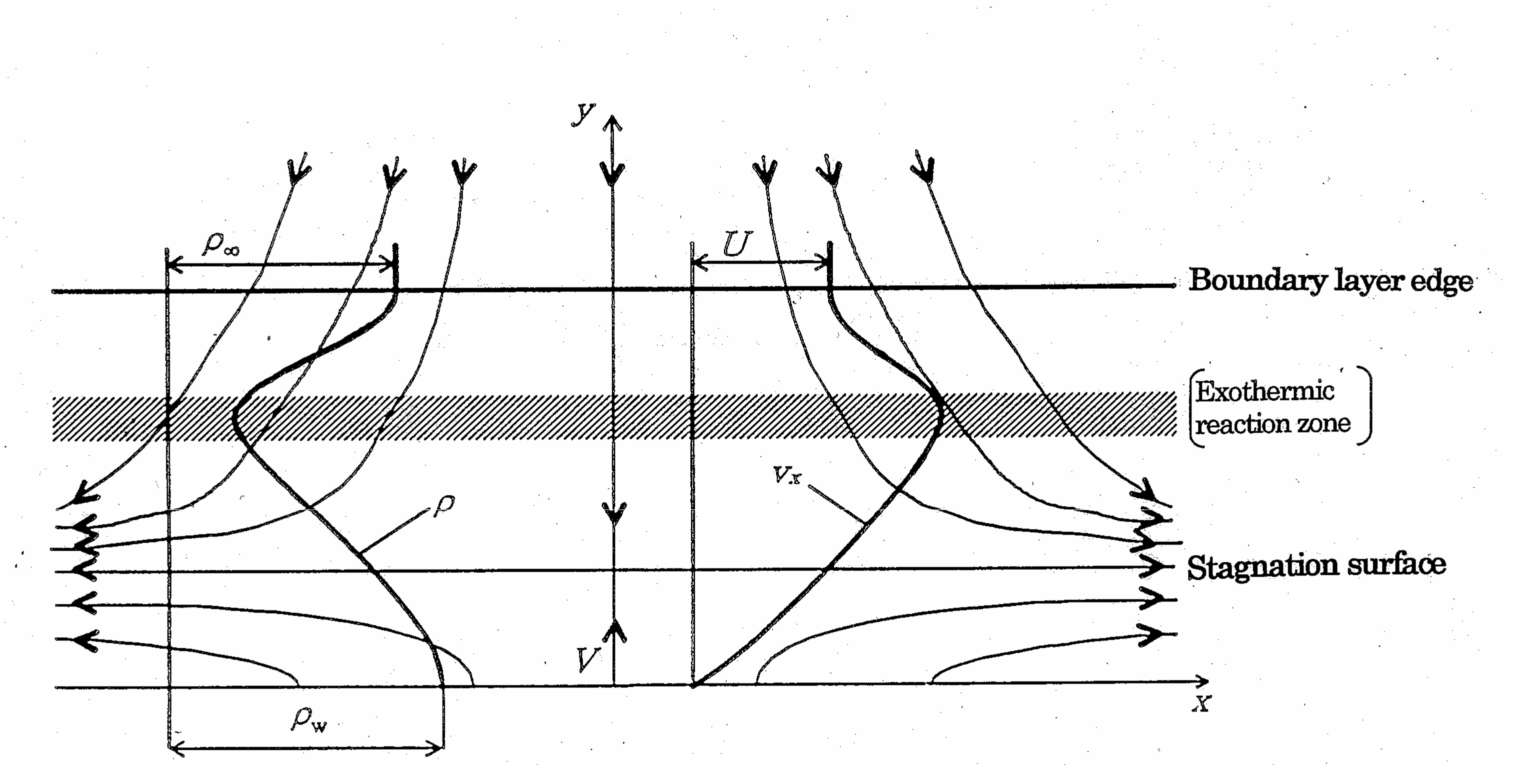

y
ρ_∞
U
Boundary layer edge
Exothermic reaction zone
v_x
ρ
Stagnation surface
V
ρ_w
x

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$$\left.\begin{aligned}&\xi=\int_0^x U\rho_\infty\mu_\infty x,\quad \eta=\frac{U\rho_\infty}{\xi^{\frac{1}{2}}}\int_0^y\frac{\rho}{\rho_\infty}\mathrm{d}y,\\&\Psi=\xi^{\frac{1}{2}}f(\eta),\quad \rho v_x=\frac{\partial\Psi}{\partial y},\quad \rho v_y=-\frac{\partial\Psi}{\partial x}\end{aligned}\right\}$$

$$2f'''+ff''-\frac{2}{U}\frac{\partial U}{\partial\xi}(f')^2+\frac{2a\xi}{\rho^2\mu_\infty U^2}=0$$

$$\left.\begin{aligned}&\eta=0:\quad f'=0,\quad f=0\\&\eta=\infty:\quad f'=1\end{aligned}\right\}$$

HIRANO


Thank you for your kind attention