The Role of Uncertainty on Risk Assessment

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Presentation outline

- Introduction
- Background
- RA Structure
- Sources of Uncertainty on RA
- Classification & Representation of U
- The Theory of Evidence
- The Measure of Evidence
- Example calculation
- Conclusions

Introduction

- Different words are related to “Uncertainty”: Inaccuracy, Vagueness, Ambiguity, Imprecision, etc.:

  often it depend on the Problem
- Problem under analysis: Simple>>>Complex
  - The length of this table: Simple
  - Weather forecaster: Simple in theory
    Complex in practice
  - The mood of my wife this evening: complex

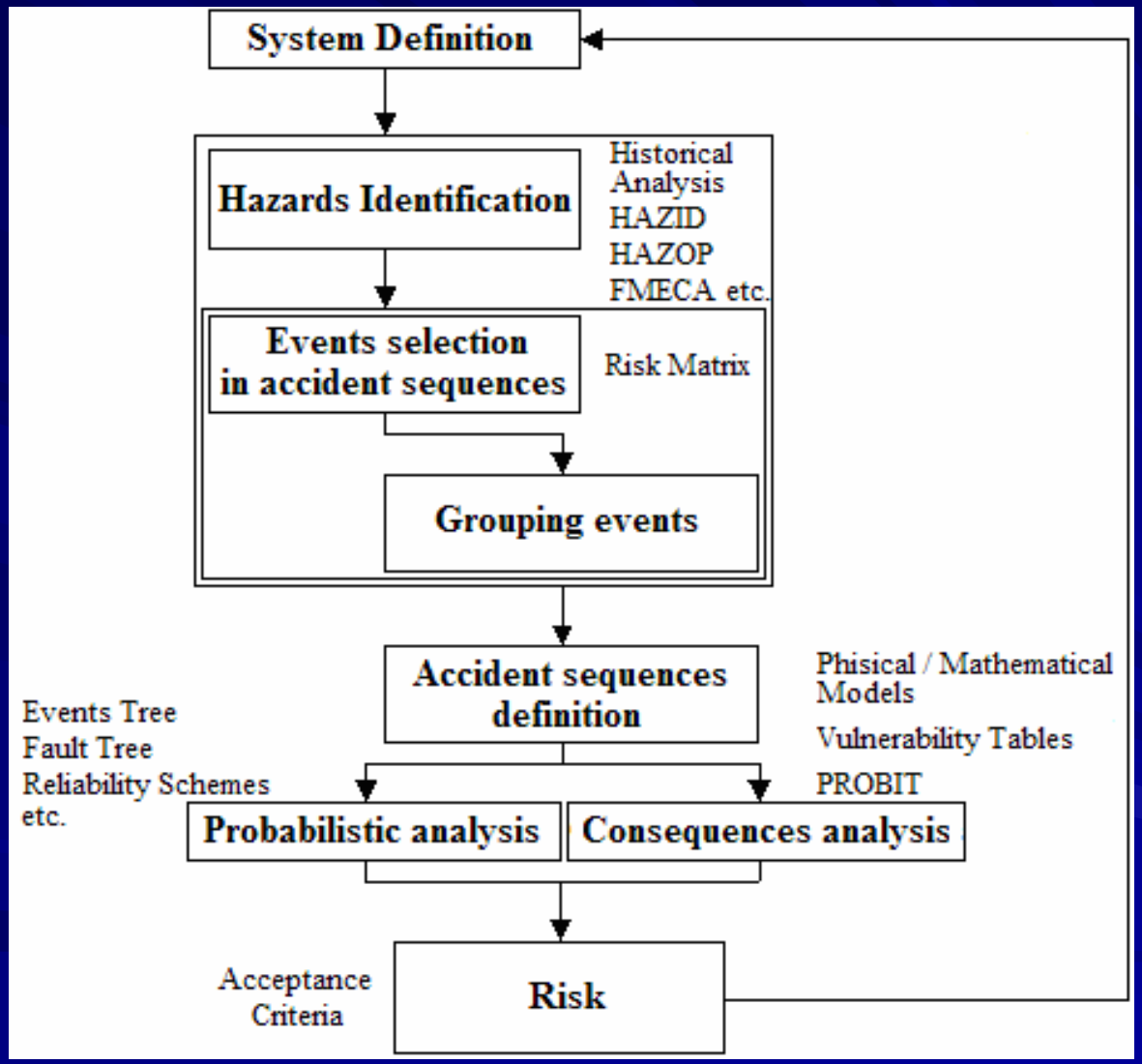
Introduction (cont)

Risk assessment is not an algorithm, but a complex cognitive process full of uncertainties

Background

- Errors and errors composition (± e; Percentile, etc.)
- Theory of Probability
- Distribution functions or Cumulative probability
- Convolution rules
- RA Background

RA Structure



RA Structure (cont)

- SYSTEM DEFINITION
- HAZARDS IDENTIFICATION
- EVENTS SELECTION
- ACCIDENTS SEQUENCES STUDY
- PROBABILISTIC ANALYSIS
- CONSEQUENCES ANALYSIS
- NUMERICAL RISK QUANTIFICATION
- ACCEPTANCE CRITERIA

Sources of Uncertainty on RA

- Imprecisely specified distributions;
- Scarcely known or even unknown dependencies;
- Non-negligible measurement Uncertainty;
- Non-detects or other censoring in measurements;
- Small sample size;
- Inconsistency in the quality of input data;
- Model Uncertainty;
- Non-stationarity (non-constant distributions).

Classification & Representation of U

The systematic study of the uncertainty requires a classification in two different typologies:

- Aleatory Uncertainty;
- Epistemic Uncertainty.

The first typology concerns the phenomena who are intrinsically stochastic. For this reason, these types of uncertainty can be described with a probabilistic approach.

The epistemic uncertainty is mainly due to incomplete knowledge of parameters and phenomena. Such “ignorance” is due to the values of Uncertainty belongs to the parameters, and to the Uncertainty of the models adopted for the description of the phenomena.

Classification & Representation of U (cont)

- Considering a phenomenon (gas explosion)
- The analysed <u>aleatory</u> variable y (e.g. probability of death)
- The function f describing the phenomenon function on n parameters/variables .
- The parameters are affected by uncertainty represented by the vector $\boldsymbol{I} = [I_1 , I_2 , \dots , I_n]$ (uncertainty of gas concentration, distance)
- Synthetically the aleatory variable $y = f(\boldsymbol{I})$
- The probability theory defines for every $\mathbf{I}_n$ a probability density function P_{In} producing a vector $\boldsymbol{P_I} = [P_{I1} , P_{I2} ,\dots , P_{In}]$.

Classification & Representation of U (cont)

- If the factors belonging to the system produce aleatory Uncertainty, the P_I distributions are enough for the description of the problem and its results have adequate precision
- It is possible to describe the problem unambiguously by using an acceptability risk assessment criterion and to define the cumulative probability distribution function as follows:

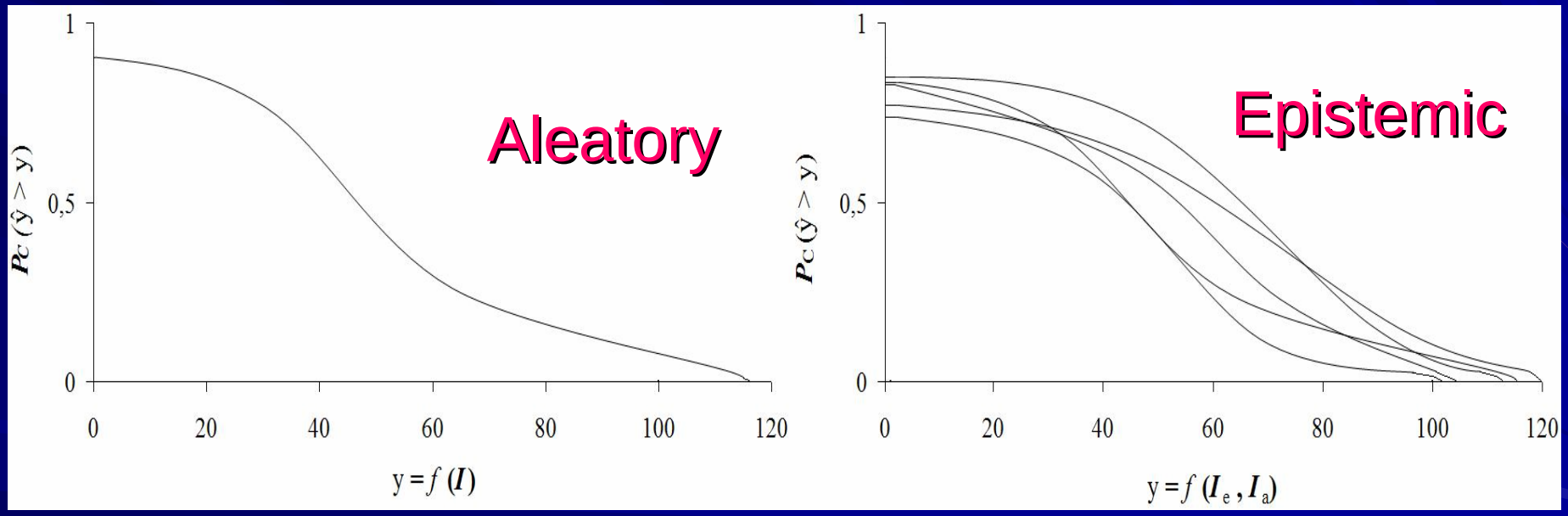
$$P_C(\hat{y} > y) = \int_I \delta[f(\boldsymbol{I})]\, P_I(\boldsymbol{I})\, d\boldsymbol{I};$$

with $\delta[f(\boldsymbol{I})] = 1$ if $f(\boldsymbol{I}) > y$;

$\delta[f(\boldsymbol{I})] = 0$ otherwise;

Classification & Representation of U (cont)

- If the Uncertainty is epistemic, it is necessary to define a new function $y = f(I_e ; I_a)$ considering the epistemic Uncertainty I_e

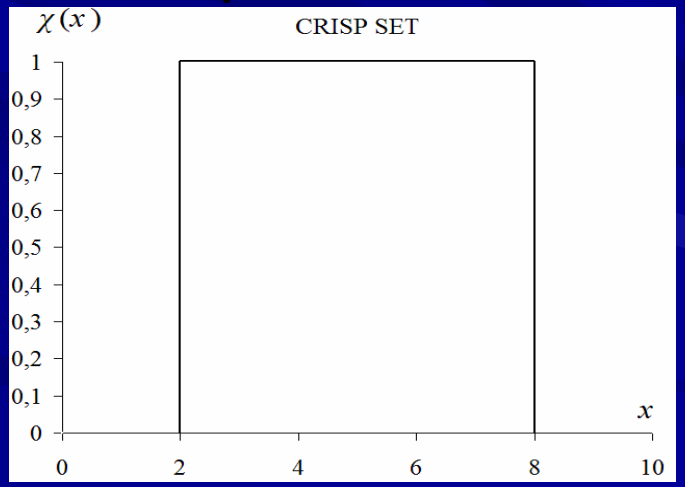


Cumulative Probability according to the kind of Uncertainty.

Classification & Representation of U (cont)

- The Fuzzy Logic Theory provides an immediate image of the Uncertainty degree associated with a variable.
- If X is the Universe of the possible events, the membership function χ of an generic element x (continuous or discrete with x_i ; i = 1, 2 …n) belonging to a classical crisp set A is defined as follows:

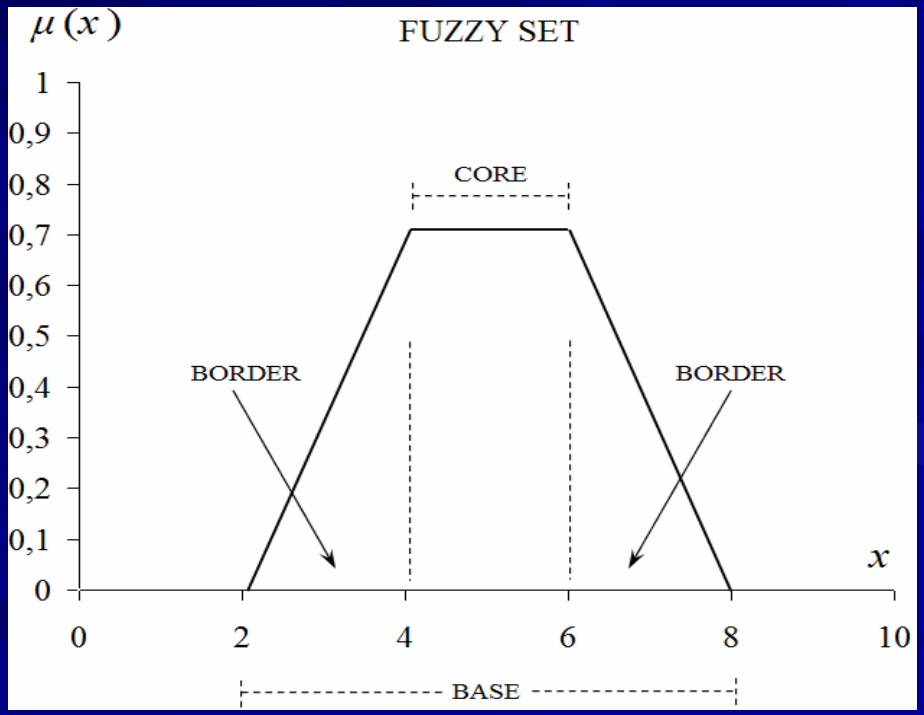
$$\chi_A(x) = \begin{cases} 0 & if \quad x \notin A \\ 1 & if \quad x \in A \end{cases}$$



Classification & Representation of U (cont)

As far as a fuzzy set $\underline{A}$ is concerned, the membership function is:

$$\mu_{\underline{A}}(x) = f(x) \qquad with \qquad f(x) \in [0,1]$$



Classification & Representation of U (cont)

- Let us considered an element x_k to correlate its membership degree $\mu_{\underline{A}}(x_k)$ in the fuzzy set $\underline{A}$ with the probability of occurrence $P(x_k)$ of the same element in the corresponding probability distribution (different from the fuzzy set). There are some general properties such as:

$$\mu_{\underline{A}}(x) = 0 \rightarrow P(x) = 0; \qquad \mu_{\underline{A}}(x) = \mu_{\underline{A}}(y) \rightarrow P(x) = P(y);$$

$$\mu_{\underline{A}}(x) > \mu_{\underline{A}}(y) \rightarrow P(x) \geq P(y).$$

- It is possible <u>to convert a fuzzy set in a probability distribution</u> using a simple normalization formula

$$P(x) = \frac{\mu(x)}{\sum \mu(x)}$$

The Theory of Evidence

All what is known (also not completely) of a phenomenon represents its evidence, all what is possible to deduce from the body of evidence to the phenomenon represents its <u>Belief</u>

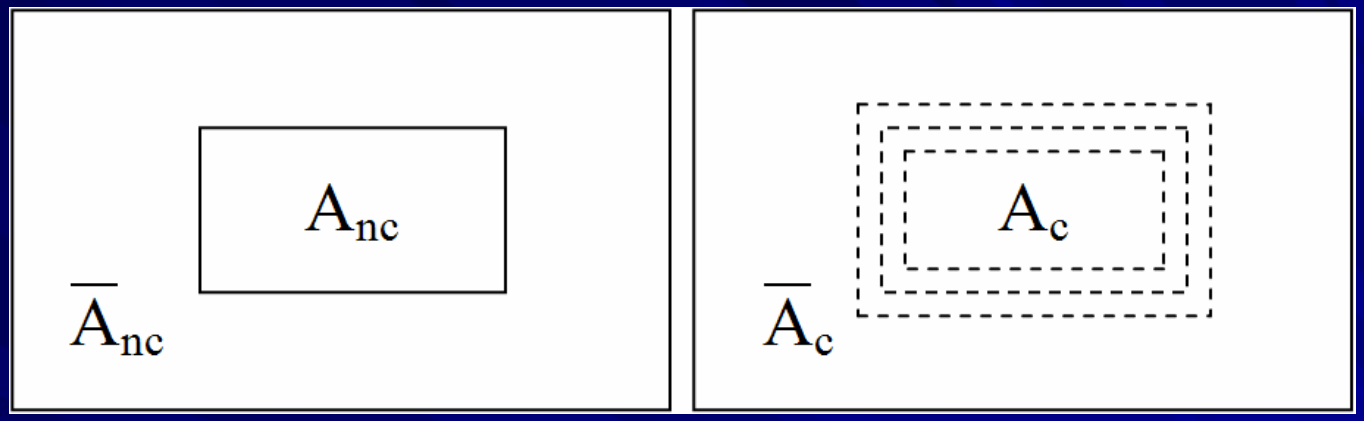
What, on the contrary, is not in contrast (induction) with the evidence of the phenomenon is identified with its <u>Plausibility</u>

$$\text{Ignorance } (A) = 1 - [\,\text{Bel}(A) + \text{Bel}(\bar{A})\,]$$

$$\text{Ignorance } (A) = [\,\text{Pl}(A) + \text{Pl}(\bar{A})\,] - 1$$

Bel (A) = Belief of the event A .
Pl (A) = Plausibility of the event A .

The Theory of Evidence (cont)



Graphic representation of a complex and vague Ac event and non complex and crisp Anc event.

When the event is not complex [Anc] the ignorance is equal to zero:

$$\mathrm{Bel}\,(A_{nc}) = \mathrm{Pl}\,(A_{nc}) = \mathrm{Pr}\,(A_{nc})$$

$$\mathrm{Pr}(A_{nc}) + \mathrm{Pr}(\overline{A}_{nc}) = 1$$

The Theory of Evidence (cont)

before becoming credible must be plausible and that, as a result, Plausibility measure is always greater or to the limit the same as that of Belief

$$\mathrm{Bel}(A) + \mathrm{Bel}(\bar{A}) \leq 1$$

$$\mathrm{Pl}(A) + \mathrm{Pl}(\bar{A}) \geq 1$$

$$\mathrm{Bel}\ (A) \leq \mathrm{Pr}\ (A) \leq \mathrm{Pl}\ (A)$$

When the Uncertainty associated with the event is null, the ignorance on the involved quantities is zero:

$$\mathrm{Bel}\ (A) = \mathrm{Pr}\ (A) = \mathrm{Pl}\ (A)$$

The Measure of Evidence

the Probability Cores are defined
through the Power Set

The Power Set is defined as the Cartesian product of the elements of a set including the null set $\varnothing$.
For example if $A \equiv [a, b, c]$ with a number An = 3 of elements, the Power Set of A called P_A , the number of elements P_{An} is equal to 2^{An}, called Focal Elements such as:

$$P_A \equiv [(\varnothing), (a), (b), (c), (a, b), (a, c), (b, c), (a, b, c)]$$

The Measure of Evidence (cont)

the Probability Cores represent the probabilistic value assigned to the evidence of the parameters of the complex problem. They constitute the knowledge, also poor, that we have of the problem.

these quantities is definite a linear application of the power set previously defined in the interval [0 ; 1]

$$m: P(X) \rightarrow [0, 1]$$

$$m(\varnothing) = 0$$

$$\sum_{A \in P(X)} m(A) = 1$$

$$Bel(A) = \sum_{B \subseteq A} m(B)$$

$$Pl(A) = \sum_{B \cap A = \varnothing} m(B)$$

The Theory of Evidence (cont)

S = sure event belonging to a nested sequence of P_X

the <u>Necessity</u> of *A* is defined

$\nu(A) = 1$ if $S \subseteq A$;
$\nu(A) = 0$ in the other cases

The <u>Possibility</u> of *A* the function:

$\pi(A) = 1$ if $A \cap S \neq \emptyset$;

$\pi(A) = 0$ in the other cases

The Theory of Evidence (cont)

The relationship between ν (A) and π (A) :

$$\nu(A) = 1 - \pi(\bar{A}) \; ; \; \pi(A) = 1 - \nu(\bar{A})$$

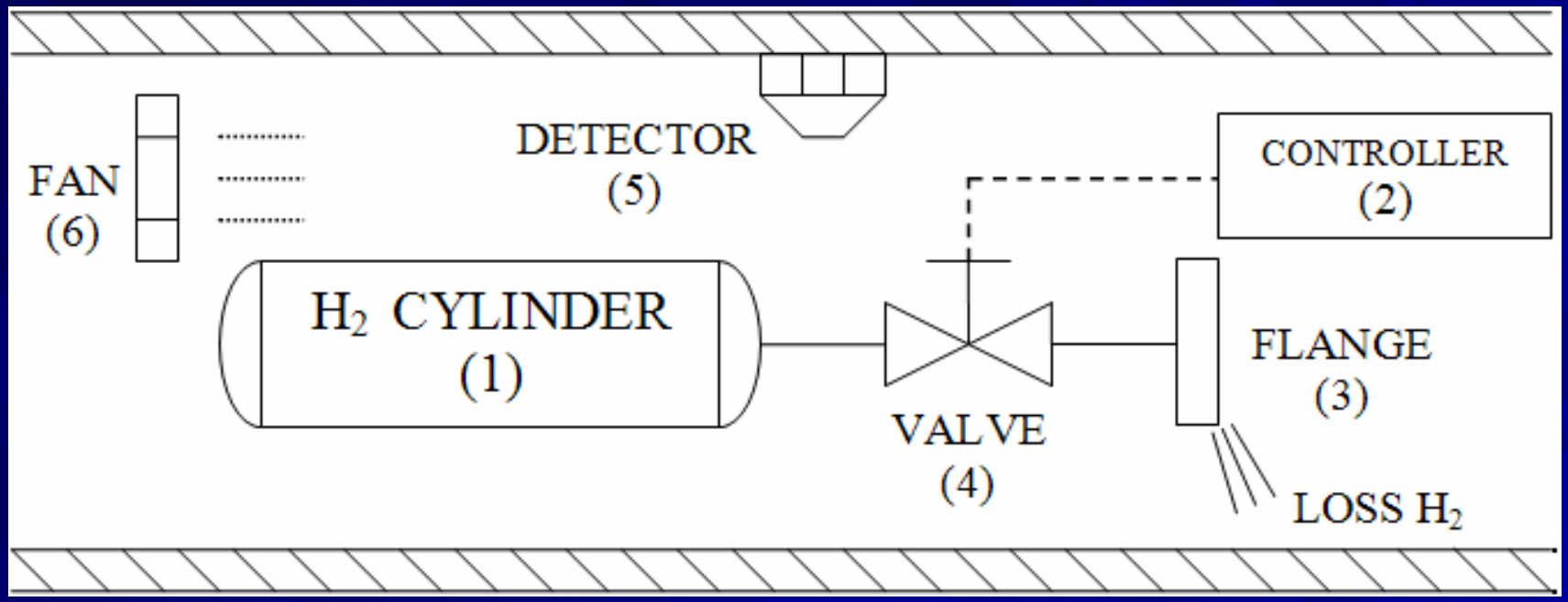
In case of not null ignorance:

$$\nu(A) \leq Pr(A) \leq \pi(A)$$

In case of ignorance = 0 :

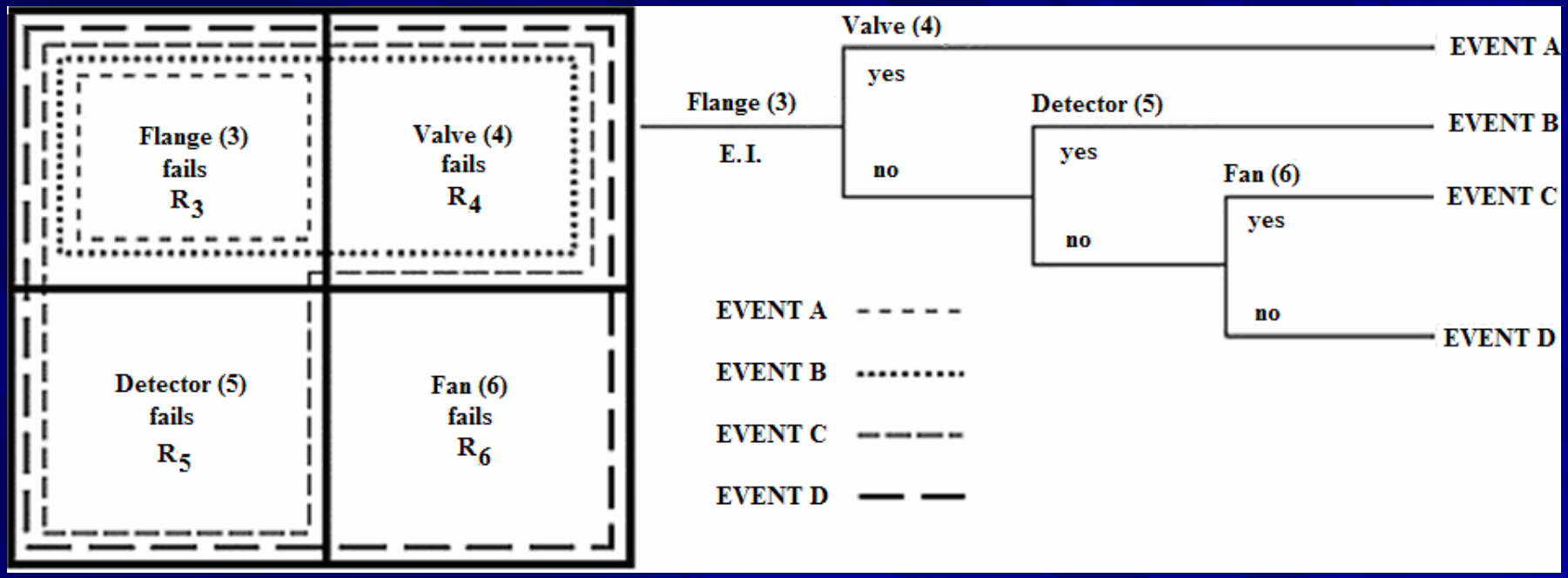
$$\nu(A) = Pr(A) = \pi(A)$$

Example calculation



Scheme of the plant:
(1) H_2 cylinder ; (2) Controller;
(3) Flange; (4) Valve;
(5) detector; (6) Fan

Example calculation (cont)

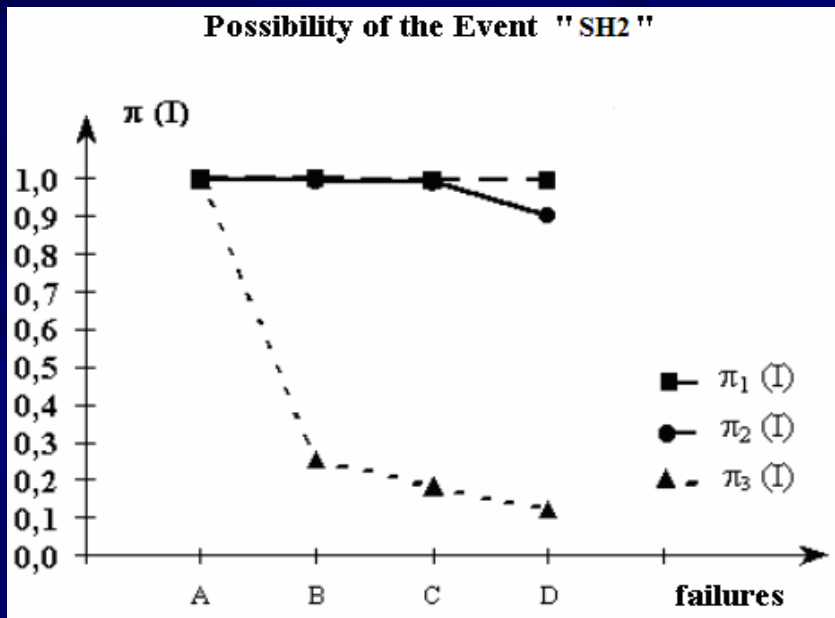


Configuration of the accidentals sequences of the events
Starting from the initiator event “failure of the flange (3)”

Example calculation (cont)

- In the Case 1 the ignorance is distributed between two events R5 (Detector fails) and R6 (Fan fails) and their combinations with the other focal elements
- In the Case 2 considers the ignorance not well identifiable in a component or reduced group of components (maximum Uncertainty degree)
- In the Case 3 a big ignorance associated with the Flange (3) behavior.

Example calculation (cont)

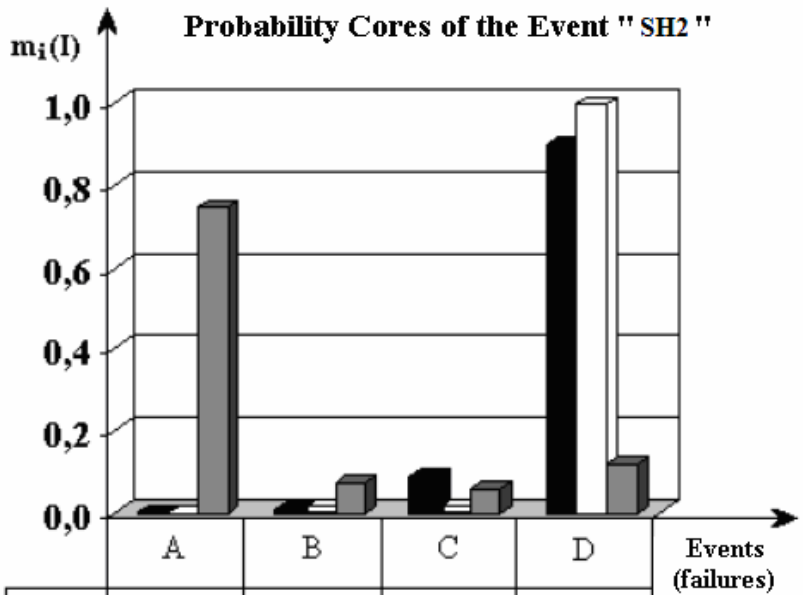


	A	B	C	D
■ m_1	0,001	0,007	0,090	0,900
□ m_2	0,001	0,002	0,003	0,994
■ m_3	0,744	0,075	0,061	0,120

Example of calculus $\pi_2(I)$

$$\pi_2(A) = 0.001 + 0.002 + 0.003 + 0.994 = 1$$
$$\pi_2(B) = 0.007 + 0.09 + 0.9 = 0.997$$
$$\pi_2(C) = 0.09 + 0.9 = 0.99$$
$$\pi_2(D) = 0.9 = 0.9$$

Example calculation (cont)



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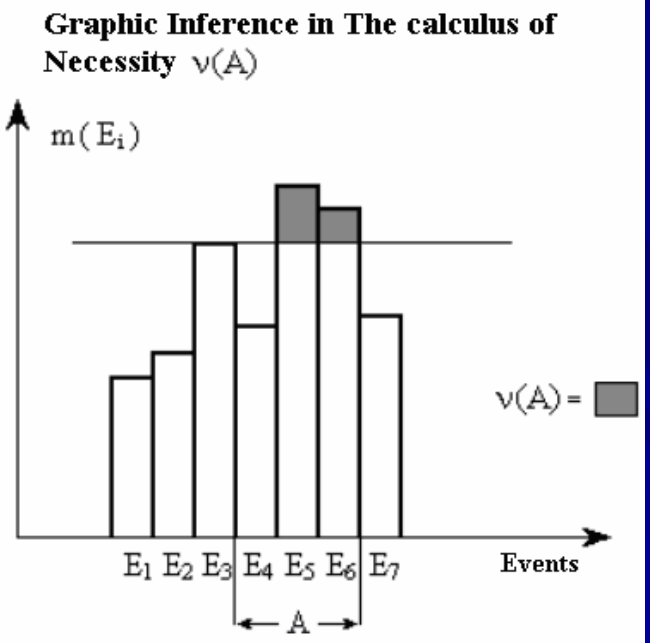
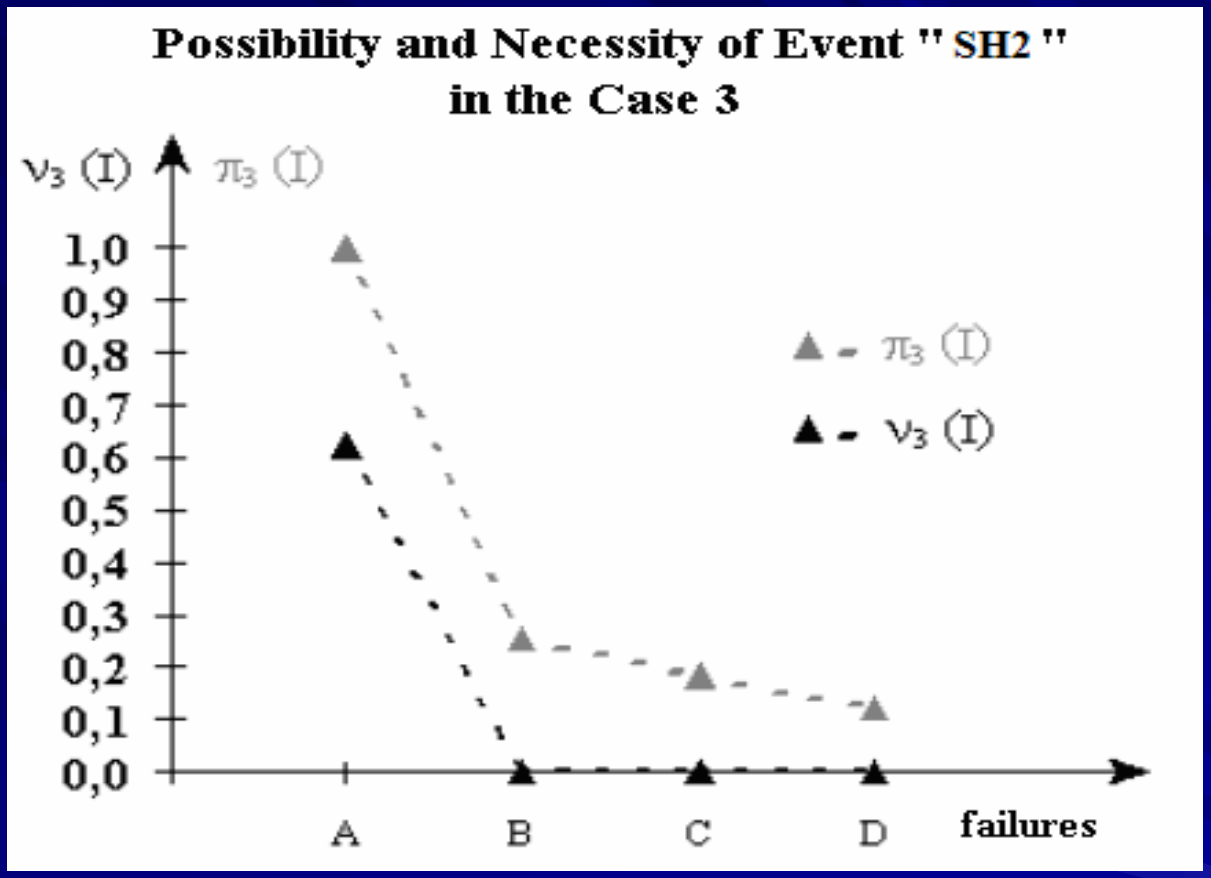


Values of the Probability Cores m (I) relative to the event SH2, used in the three considered cases and graphic inference for the calculation of the Necessity Distribution

Example calculation (cont)



Possibility and Necessity Distributions of the event SH2 versus (A,B,C,D) failures in the Case 3.

Conclusions

- the theory can be used, as analysis tool, for the identification of the critical members of a system;
- The application of Theory of Evidence, discussed in this paper, shows in detail how the expert plays a crucial role in the building of the model;

Conclusions (cont)

- Considering the most extreme values possible for a quantity, given the smallness of the data sets typically available to analysts, the chances that they are representative of the distribution are few.
- It may not always be obvious what measurement Uncertainty is associated with a particular measurement. This is particularly true for historical data.

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THANK YOU FOR YOUR ATTENTION

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